

# CHARTERED ACCOUNTANCY - BUSINESS LEVEL I

## BUSINESS MATHEMATICS AND STATISTICS.

### REGRESSION AND CORRELATION.

1. For a given set of data,  $\sum x = 680$ ,  $\sum y = 996$ ,  $\sum x^2 = 20154$ ,  $\sum y^2 = 34670$ ,  $\sum xy = 24844$ ,  $n=30$ . Find the product moment correlation coefficient.
2. By using the above data, find the equation of the regression line of  $y$  on  $x$ .
- ③ For a given set of data  $\sum x = 74$ ,  $\sum y = 4$ ,  $\sum x^2 = 820$ ,  $\sum y^2 = 44$ ,  $\sum xy = 81$ ,  $n=8$ . Find the product moment correlation coefficient.
- ④ By using the above data, find the equation of the regression line of  $y$  on  $x$ .
- ⑤ For a given set of data,  $\sum x = 108$ ,  $\sum y = 625$ ,  $\sum x^2 = 1740$ ,  $\sum xy = 9356$ ,  $n=8$ . Find the equation of the regression line of  $y$  on  $x$ .
- ⑥ For a given set of data  $\sum x = 557$ ,  $\sum y = 871$ ,  $\sum xy = 50206$ ,  $\sum x^2 = 32389$ ,  $\sum y^2 = 79141$ ,  $n=10$ . Find the product moment correlation coefficient.
- ⑦ Calculate the product moment correlation coefficient by using the data given below

$x$	1	2	3	4
$y$	3	8	11	14

- ⑧ Find the equation of the regression line of  $y$  on  $x$  by using the data given below.  
 $n=5$ ,  $\sum x = 25$ ,  $\sum y = 100$ ,  $\sum xy = 570$ ,  $\sum x^2 = 145$

(9) Find the product moment correlation coefficient between  $x$  and  $y$  by using the data given below.

$x$	1	2	3	4	5
$y$	8	6	4	2	0

(10) If  $r = 0.9$ , find the coefficient of determination.

(11) Find the equation of the regression line of  $y$  on  $x$  by using the data given below

$$n=5, \sum x = 100, \sum y = 400, \sum xy = 8104, \sum x^2 = 2040.$$

(12) The following values have been calculated by using 10 pairs of observations. Find the equation of the regression line of  $y$  on  $x$ ,

$$\sum x = 65 \quad \sum y = 61 \quad \sum xy = 464 \quad \sum x^2 = 475.$$

(13) Find the equation of the regression line of  $y$  on  $x$  by using the following data.

$$n=10, \sum x = 500, \sum y = 300, \sum xy = 16837, \sum x^2 = 27818$$

(14) Complete the table given below and (i) find the equation of the regression line of  $y$  on  $x$  (ii) calculate P.M.C.

$x$	$y$	$xy$	$x^2$	$y^2$
2	5	10	4	25
4	7	28	16	
6	9			
8	8			
10	11	110	100	121

ANSWERS: (1)  $r = 0.82$  (2)  $y = 22.36 + 0.48x$ , (3)  $r = 0.58$

(4)  $y = -2.5 + 0.32x$ . (5)  $y = 34.93 + 3.21x$ , (6)  $r = 0.847$

(7)  $r = 1$  (8)  $y = 2.5 + 3.5x$ , (9)  $r = -1$  (10)  $0.81$

(11)  $y = 2.8 + 2.6x$  (12)  $y = -2.29 + 1.29x$  (13)  $y = 0.65x - 2.59$ .

(14) (i)  $y = 4 + 0.65x$ .

## CORRELATION

When the value of one variable is related to the value of another, they are said to be correlated. Thus, correlation means an interrelationship or association. For example,

- there is likely to be some correlation between a person's height and weight.
- there is a high correlation between age and height of the children upto a certain age.

The degree of correlation between two variables can be measured and there are two measures of correlation.

(a) The Product Moment Correlation Coefficient.

(b) The Rank Correlation coefficient.

(a) PEARSON'S PRODUCT MOMENT CORRELATION COEFFICIENT (PMCC)

Pearson's Product Moment Correlation coefficient is denoted by  $r$  and  $r$  is defined as,

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \times \sum (y - \bar{y})^2}}$$

$$\bar{x} = \frac{\sum x}{n} \quad \text{and} \quad \bar{y} = \frac{\sum y}{n}$$

This formula can be written as

$$r = \frac{n \sum xy - \sum x \cdot \sum y}{\sqrt{\{n \sum x^2 - (\sum x)^2\} \{n \sum y^2 - (\sum y)^2\}}}$$

$n$  is the number of pairs of variables

$r$  lies between  $-1$  and  $+1$ , i.e.  $-1 \leq r \leq 1$

POSITIVE OR DIRECT CORRELATION.

Correlation is positive or direct when increase in variables of one series is correlated with increase in corresponding variables of the other series.

Example : Machine maintenance cost and age.

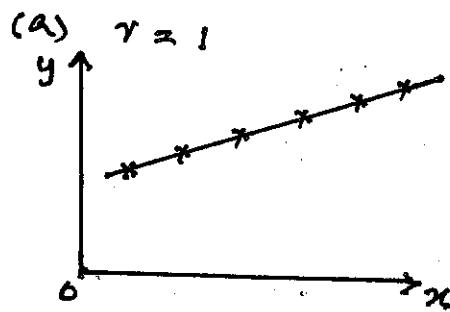
## NEGATIVE OR INVERSE CORRELATION.

Correlation is said to be negative or inverse, if increase of one is related to decrease of the other.

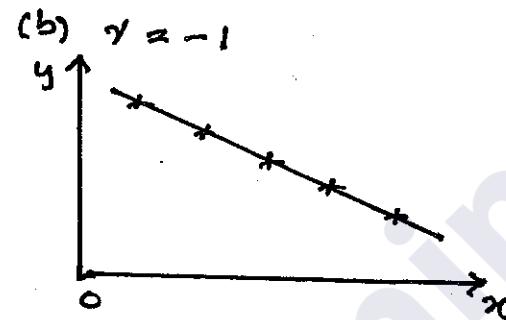
Example : Amount of goods sold and price per unit.

### DIAGRAMATIC DEMONSTRATION OF CORRELATION.

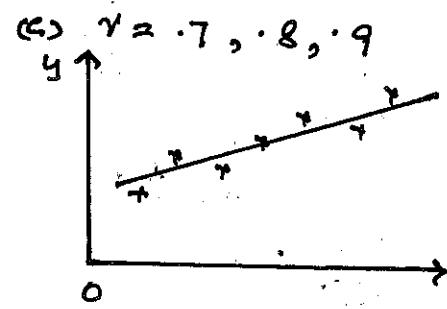
(Interpretation of the value of  $r$ .)



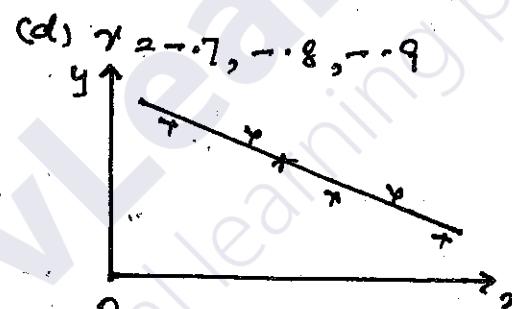
There is a perfect positive correlation between  $x + y$ .



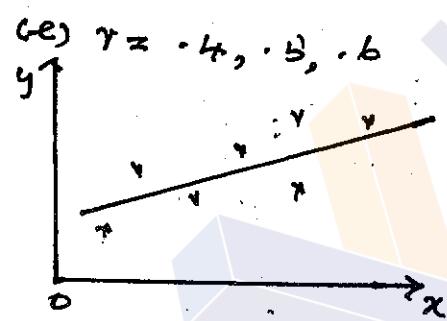
There is a perfect negative correlation between  $x + y$ .



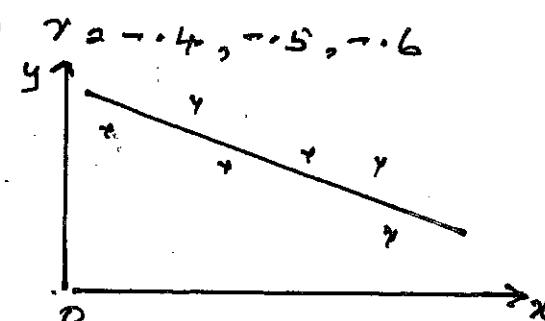
There is a high positive correlation between  $x + y$ .



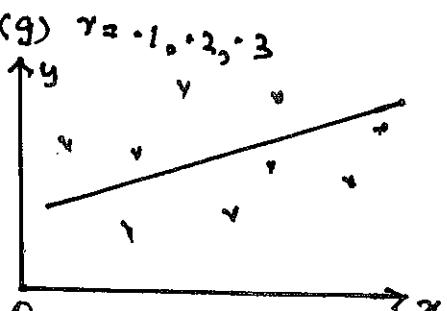
There is a high negative correlation between  $x + y$ .



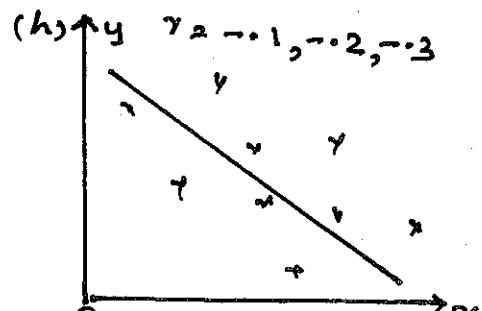
There is some positive correlation between  $x$  and  $y$ .



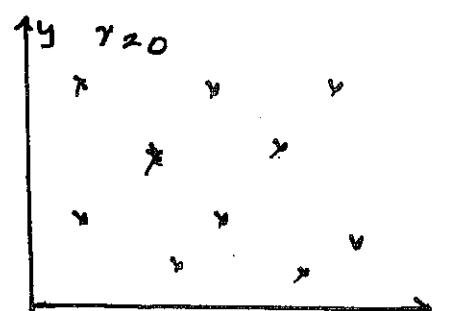
There is some negative correlation between  $x$  and  $y$ .



There is a weak positive correlation between  $x + y$ .



There is a weak negative correlation between  $x + y$ .



There is no correlation between  $x + y$ .

CHARTERED ACCOUNTANCY - BUSINESS MATHS AND STATISTICS.

Q1. The advertising cost and sales income of a company for 5 months are given below.

Ad. cost (Rs mn) (x) 6 2 10 4 8

Sales income (Rs mn) (y) 9 5 11 8 7

(i) Find the two normal equations that should be solved to find out a and b of the regression equation of y on x.

(ii) Find the equation of the regression line of y on x.

(iii) Estimate the sales income when the ad. cost is Rs 9 mn.

(Ans: Find the correlation coefficient between x and y.)

Q2. For a given set of data,  $\sum x = 38$ ,  $\sum y = 89$ ,

$\sum xy = 495$ ,  $\sum x^2 = 270$ ,  $\sum y^2 = 1147$ ,  $n=7$

Find the equation of the regression line of y on x.

(Ans:  $y = 0.186x + 11.7$ .)

Q3. For a given set of data,  $\sum x = 620$ ,  $\sum y = 996$

$\sum xy = 24844$ ,  $\sum x^2 = 20154$ ,  $\sum y^2 = 34670$ ,  $n=30$ .

Find the correlation coefficient between x and y.

(Ans:  $r = 0.82$ )

Q4-

x	y
0	4
12	7
8	9
14	11
3	12
8	16

$$\sum xy = 634, \sum x^2 = 737, \sum y^2 = 667$$

Calculate the product moment correlation coefficient between x and y by using the given data.

(Ans:  $r = 0.655$ ).

05. 80 people were asked to measure their pulse rates when they woke up in the morning. The mean was 69 beats and s.d. 4 beats. Find 95% c.l. for the population mean. (Ans: 68.12, 69.88).
06. In a survey carried out in a large city, 170 households out of a random sample of 250 owned at least one pet. Find 95% c.l. for the proportion of households in the city who own at least one pet. (Ans: 0.622, 0.788).
07. The weights of steel sheets produced by a plant are known to be normally distributed with mean 31.4 kg and s.d. 2.4 kg. Find the percentage of sheets that weigh more than 35.6 kg. (Ans: 4.01%).
08. The lifetime of a torch battery has a normal distribution with mean 210 hrs and s.d. 12 hrs. Find the probability that a torch battery selected at random will last between 205 hrs and 215 hrs. (Ans: 0.3231).
09. A discrete random variable has the probability distribution shown in the table below.
- | $x$    | 1   | 2   | 3   | 4   |
|--------|-----|-----|-----|-----|
| $P(x)$ | 0.2 | 0.3 | 0.4 | 0.1 |
- Calculate (i)  $P(2 < x \leq 3)$  (ii) Mean (iii) Variance.  
 (Ans: 0.7, 2.4, 0.84).
10. In a particular mixed school 60% are boys and 40% are girls. 40% of the boys play tennis and 30% of girls play tennis. A pupil is selected at random. find the probability that  
 (i) the pupil is a girl who plays tennis  
 (ii) the pupil plays tennis  
 (iii) the pupil is a girl, given that the pupil plays tennis  
 (Ans: 0.12, 0.36, 0.33).