Chapter 3

FINANCIAL MATHEMATICS - I

* Simple Interest
* Compound Interest
* Regular Investments

INTEREST AMOUNT

Interest is amount of money earned by an investment over a given period of time.

SIMPLE INTEREST

When interest is calculated on the originally invested amount for every period, it is known as Simple Interest.

Example:-

A person invested Rs. 10,000 under 10% annual Simple Interest rate.

* End of Year I

Interest Amount	Total Amount
$I_1 = 10{,}000 \times \frac{10}{100} \times 1$	$S_1 = 10{,}000 + 1{,}000$
I_1 = Rs. 1,000	S_1 = Rs. 11,000

* End of Year II

Interest Amount	Total Amount
$I_2 = 10{,}000 \times \frac{10}{100} \times 2$	$S_2 = 10{,}000 + 2{,}000$
I_2 = Rs. 2,000	S_2 = Rs. 12,000

* End of Year III

Interest Amount	Total Amount
$I_3 = 10{,}000 \times \frac{10}{100} \times 3$	$S_3 = 10{,}000 + 3{,}000$
I_3 = Rs. 3,000	S_3 = Rs. 13,000

Formulae for Simple Interest

Let the originally invested amount. Which is usually known as principal be 'X', Let the interest rate as r% per annum, Let the number of periods as 'n'.

The simple interest is given by:

$$I = Xnr$$

At the end of time period the investment has total amount 'S' is given by:

$$S = X + I$$
$$S = X + Xnr$$
$$S = X (1+nr)$$

Worked Examples

Example ①

Calculate total amount and interest earned from following investments.

(a) Invested Rs. 50,000 under 8% annual simple interest rate for 4 years
(b) Invested Rs. 40, 000 under 2.5% monthly simple interest rate for 10 months.
(c) Invested Rs. 75,000 under 12% annual simple interest rate for 6 months.
(d) Invested Rs. 120,000 under 3% monthly simple interest rate for $\frac{1}{2}$ years.

Solution

(a) $S = X + Xnr$
$X = Rs.\ 50,000$
$r = 0.08$ (annual)
$n = 4$ years

$S = 50,000 + 50,000 \times 0.08 \times 4$
$S = 50,000 + 16,000$
$S = Rs.\ 66,000$

$I = Rs.\ 16,000$

(b) S = X + Xnr
X = Rs. 40,000
r = 0.025 (monthly)
n = 10 months

S = 40,000 + 40,000 × 0.025 × 10
S = 40,000 + 10,000
S = Rs. 50,000

I = Rs. 10,000

(c) S = X + Xnr
X = Rs. 75,000
r = 0.012 (annual)
n = $6/12 = 1/2$ years

S = 75,000 + 75,000 × 0.012 × $1/2$
S = 75,000 + 4,500
S = Rs. 79,500

I = Rs. 4,500

(d) S = X + Xnr
X = Rs. 120,000
r = 0.03 (monthly)
n = 1.5 × 12 = 18 months

S = 120,000 + 120,000 × 0.03 × 18
S = 120,000 + 64,800
S = Rs. 184,800

I = Rs. 64,800

Exercise 01

Identify the better investment in terms of Interest per annum Rs. 50,000 invested 8% per annum or Rs. 60,000 invested at 7% per annum.

Exercise 02

Compute the simple interest that would be earn on an investment of Rs. 36,000 at 4% per month over 3 years.

Example ②

Calculate the period necessary for an investment to reach Rs. 798,000, if the original value is Rs. 700,000 and rate of invest 7% per annum.

Solution

S = X + Xnr
X = Rs. 700,000
S = Rs. 798,000
r = 0.07 (annual)
n = ? (years)

$798,000 = 700,000 + 700,000 \times 0.07 \times n$
$798,000 = 700,000 + 49,000n$
$98,000 = 49,000n$
$\frac{98,000}{49,000} = n$

n = 2 years

Exercise 03

In four years an investment of Rs. 900,000 has grown 1,548,000. Calculate monthly rate of interest.

Example ③

An investor receives Rs. 1,160,000 as annual returns from two investments earning 3% and 5% simple interest. He invests 25% more in first investment which earns 3% interest and 40% more in the second investment which earns 5% by doing this he would be able to increase his annual returns of Rs. 410,000 more. Calculate initial value of the investments.

Solution

Let the amount to be invested under 3% = X
5% = Y

First Investment

$1,160,000 = 0.03X + 0.05Y$

Second Investment

1,160,000 + 410,000 = 1.25X (0.03) + 1.40Y (0.05)
1,570,000 = 0.0375X + 0.07Y

0.03X + 0.05Y = 1,160,000 ⟶ ①
0.375X + 0.07Y = 1,500,000 ⟶ ②

By using calculator

$a_1 = 0.03$ $a_2 = 0.0375$
$b_1 = 0.05$ $b_2 = 0.07$
$c_1 = 1,160,000$ $c_2 = 1,570,000$

X = Rs. 12,000,000
Y = Rs. 16,000,000

Exercise 04

A person invested money at annual rates of interest of 5% and 8% last year and received Rs.30,000 as interest. This year he tripled the amount invested at 8% and will receive Rs. 78,000 as an interest. Calculate amount invested under each rate.

COMPOUND INTEREST

Compound Interest is an interest that is paid on both principal and also any interest from past period.

Example

A person invested Rs. 10,000 under 10% annual interest rate compounded annually.

A person invested Rs. 10,000 under 10% annual Simple Interest rate.

* End of Year I

Interest Amount	Total Amount
$I_1 = 10{,}000 \times \frac{10}{100} \times 1$	$S_1 = 10{,}000 + 1{,}000$
I_1 = Rs. 1,000	S_1 = Rs. 11,000

* End of Year II

Interest Amount	Total Amount
$I_2 = 10{,}000 \times \frac{10}{100} \times 1$	$S_2 = 10{,}000 + 1{,}100$
I_2 = Rs. 1,100	S_2 = Rs. 12,100

* End of Year III

Interest Amount	Total Amount
$I_3 = 10{,}000 \times \frac{10}{100} \times 1$	$S_3 = 12{,}100 + 1{,}210$
I_3 = Rs. 1,210	S_3 = Rs. 13,310

Compound Interest Formulae

$$S = X\left(1+\frac{r}{m}\right)^{mn}$$

* X = Original Sum Invested
* S = Total amount of money at the end of time period
* r = Annual interest rate
* n = No. of years
* m = No. of compounding period per year

* If compounded Annually (m = 1)

  $S = X (1 + r)^{n}$

* If compounded Semi-Annually (m = 2)

  $S = X \left(1+\frac{r}{2}\right)^{2n}$

* If compounded Quarterly (m = 4)

  $S = X \left(1+\frac{r}{4}\right)^{4n}$

* If compounded Monthly (m = 12)

  $S = X \left(1+\frac{r}{12}\right)^{12n}$

Worked Examples

Example ①

Calculate total amount and interest earned from following investments.

Invested Rs. 100,000 under 12% annual interest rate for 5 years if compounded
(a) Annually
(b) Semi - Annually
(c) Quarterly
(d) Monthly

Solution

(a) Annually

$S = X (1 + r)^{n}$
X = Rs. 100,000
r = 0.12 (annual)
n = 5 years

$S = 100,000 (1 + 0.12)^{5}$
$S = 100,000 (1.12)^{5}$
S = Rs. 176,274

(b) Semi - Annually

$S = X\left(1+\frac{r}{2}\right)^{20}$

X = Rs. 100,000
r = 0.12/2 = 0.06 (Semi - annual - rate)
2n = 2 × 5 = 10 (Semi - annual periods)

$S = 100,000\ (1 + 0.06)^{10}$
$S = 100,000\ (1.06)^{10}$
S = Rs. 179,085

(c) Quarterly

$S = X\left(1+\frac{r}{4}\right)^{4n}$

X = Rs. 100,000
r/4 = 0.12/4 = 0.03 (Quarterly - rate)
4n = 4 × 5 = 20 (Quarters)

$S = 100,000\ (1 + 0.03)^{20}$
$S = 100,000\ (1.03)^{20}$
S = Rs. 180,611

(d) Monthly

$S = X\left(1+\frac{r}{12}\right)^{12n}$

X = Rs. 100,000
r/12 = 0.12/12 = 0.01 (Monthly rate)
12n = 12 × 5 = 60 (Months)

$S = 100,000\ (1 + 0.01)^{60}$
$S = 100,000\ (1.01)^{60}$
S = Rs. 181,670

Exercise 01

Ravi invests Rs. 700 on 1st of January each year starting in 2010. Compound interest of 10% was credited on 31st December each year. To the nearest rupee calculate the credit of his investment on 31st December 2019.

Example ②

Rs. 2,500 invested on 1st of January 1999 had grown to be worth Rs. 61,482 on 31st December 2013. Calculate the equivalent annual compound growth rate to one decimal place.

Solution

S = X $(1 + r)^n$
X = Rs. 2,500
S = Rs. 61,482
n = 15 years
r = ? (annual)

$61{,}482 = 2{,}500\ (1 + r)^{15}$

$\frac{61{,}482}{2{,}500} = (1 + r)^{15}$

$24.5982 = (1 + r)^{15}$

$\sqrt[15]{24.5982} = 1 + r$

$1.238 = 1 + r$

$0.238 = r$

$23.8\% = r$

Exercise 02

After 15 years an investment of Rs. 60,000 had grown to Rs. 674,000. Calculate the annual rate of compound interest that has been applied to one decimal place.

Exercise 03

A company borrowed Rs. 8,000,000 at a compound interest rate of 6% per annum after some time company pay back the debt, and obtained the amount accrued up to Rs. 11,348,153. Calculate the period during which the debt remained un-settled, if there were no re-payments during the period.

Example ③

An item of equipment currently costs Rs. 4,000,000. The rate of inflation for 3 years is expected to be 8% per annum, then 10% per annum for the following 2 years. The price of an equipment is expected to increase in line with the inflation. Compute the price nearest Rs. 000 after 5 years.

Solution

Price of an equipment after 3 years.

S = X $(1 + r)^n$
X = Rs. 4,000,000
r = 0.08 (annual)
n = 3 years
S = S_1 = ?

S = 4,000,000 $(1 + 0.08)^3$
S = 4,000,000 $(1.08)^3$
S = Rs.5,038,848

Price of an equipment after next 2 years.

S = X $(1 + r)^n$
X = S_1 = Rs. 5,038,848
r = 0.1 (annual)
n = 2 years
S = ?

S = 5,038,848 $(1 + 0.1)^3$
S = 5,038,848 $(1.1)^3$
S = Rs.6,097,006
S = Rs. 6,097,000

Exercise 04

A person deposits Rs. 100,000 for 10 years. The compound rate of interest earned is 6% for the first 4 years and 5% last 6 years. Compute the value of investment to nearest rupee.

REGULAR INVESTMENTS

A person who invests money at regular time intervals with equal installments, then it is said to be a regular investment.

The amount available of a regular investment at the end of a given period can be calculated using the equations given below.

Case I (where deposits are made at the end of time interval)

$$S = \frac{A\{R^n - 1\}}{R - 1}$$

Case II (where deposits are made at the beginning of time interval)

$$S = \frac{AR\{R^n - 1\}}{R - 1}$$

* S = Total amount of the end of time period
* A = Size of an installment
* n = Time period

R = 1 + r
r = Rate of interest

Sinking Fund

When an amount of money will be needed some future date. It is good practice to accumulate systematically a fund that will become equal to required amount at the time it is needed. The money accumulate this way is called sinking fund.

Worked Examples

Example ①

Mr. Perera invested 12 annual payments of Rs. 20,000 into an investment fund earning a compound interest of 6% per annum. Calculate the value of the fund at year 12 if the first payment was at year zero.

Solution

This is a regular investment, which deposits money at beginning of time interval.

$$S = \frac{AR\{R^n - 1\}}{R - 1}$$

A = Rs. 20,000
n = 12
r = 0.06
R = 1.06

$$S = \frac{20{,}000(1.06)\{(1.06)^{12} - 1\}}{1.06 - 1}$$

$$S = \frac{21{,}200\{2.01 - 1\}}{0.06}$$

$$S = \frac{21{,}200 \times 1.01}{0.06}$$

S = Rs. 356,867

Exercise 01

A person sets aside Rs. 80,000 at the end of every year in order for 10 years to create a sinking fund. Compute amount available at the end of 10 years, if the interest rate is 5% per annum.

Example ②

A company's cricket club sets up a reserve fund so that its equipment can be renewed in exactly 5 years from now, at an expected cost of Rs. 1,000,000. Calculate the amount that should be deposited into the fund at the beginning of each of the five years if compound interest at 9% per annum can be obtained.

Solution

This is a regular investment, where annual amounts are deposited at the beginning of each year.

$$S = \frac{AR\{R^n - 1\}}{R - 1}$$

S = Rs. 1,000,000
n = 5
r = 0.09
R = 1.09

$$1{,}000{,}000 = \frac{A(1.09)\{(1.09)^5 - 1\}}{1.09 - 1}$$

$$1{,}000{,}000 = \frac{A(1.09)\{1.54 - 1\}}{0.09}$$

$$1{,}000{,}000 = \frac{A(1.09)\times(0.54)}{0.09}$$

$$1{,}000{,}000 = 6.54A$$

$$\frac{1{,}000{,}000}{6.54} = A$$

$$\text{Rs. } 152{,}905 = A$$

Exercise 02

Suppose a machine costing Rs. 80,000 is to be replaced at the end of 8 years, at which it will have a salvage value of Rs. 1,200. In order to provide money at that time for a new machine costing the same amount, a sinking fund can be set-up. If equal payments are placed in the fund earn 14%compound annually, what should each payment be? (Assume that deposits are placed in an account at the end of each year)

Exercise 03

(a) In three years from now (31.12.2017) a company will replace its company cars. It is estimated that Rs. 160 million will then be required to provide for this, Rs. X million is allocated now, invested at 8% per year with interest compounded every quarter. Calculate value of X.

(b) Assume that the amount required in (a) is not available, but the company can put Rs. 10 million into a reserve fund every quarter starting on 01.01.2015. Annual interest is 8% compounded every quarter. Calculate the shortfall of the target of Rs. 160 million on 31.12.2017.

(c) Calculate the amount that should be set aside each quarter starting on 01.01.2015 to ensure that Rs. 160 million will be available on 31.12.2017.

Effective Rate of Interest

(Actual Percentage Rate)

When rate of interest is r% per annum compounded m times per year, 'r' is called the nominal rate of interest. We can find the effective rate of interest that will give the same amount of interest as earlier when compounded once a year. It is given by the following formula.

$$APR = \left(1 + \frac{r}{m}\right)^m - 1$$

* APR = Actual Percentage Rate
* r = Annual Interest Rate
* m = No. of compounding periods per year.

Worked Example

Example ①

Find the Actual Percentage rate for a nominal rate of 18% per annum if compounded

(a) Semi - Annually
(b) Quarterly
(c) Monthly

Solution

(a) Semi - Annually

$$APR = \left(1+ {}^{r}\!/_{m}\right)^{m} - 1$$

r = 0.18
m = 2

$$APR = \left(1+\frac{0.18}{2}\right)^{2} - 1$$

$APR = (1+0.09)^2-1$
$APR = (1.09)^2-1$
$APR = 0.1881$
$APR = 18.81\%$

(b) Quarterly

$$APR = \left(1+ {}^{r}\!/_{m}\right)^{m} - 1$$

r = 0.18
m = 4

$$APR = \left(1+\frac{0.18}{4}\right)^{4} - 1$$

$APR = (1+0.045)^4-1$
$APR = 0.1925$
$APR = 19.25\%$

(c) Monthly

$APR = \left(1 + r/m\right)^m - 1$

$r = 0.18$

$m = 12$

$APR = \left(1 + \frac{0.18}{12}\right)^{12} - 1$

$APR = (1+0.015)^{12}-1$

$APR = (1.015)^{12}-1$

$APR = 0.1956$

$APR = 19.56\%$

Exercise 01

Calculate Actual Percentage rate of,

(a) 1.25% per month, compound

(b) 3.50% per quarter compound

(c) 4.25% par semi-annum compound

Withdrawals of Capital or Interest

If a person invests money in an investment and withdraws part of it whenever the person requires funds, then the amount withdrawn will cease to earn interest

Worked Example

Example ①

If a person deposits Rs. 800,000 into a bank savings account which pays interest at 10% per annum and makes no withdrawals expect at the end of year 2 and 3, when he takes out Rs.500,000 and Rs. 400,000 respectively, what would be the account balance in his account after 4 years.

Solution

Prepare a table as follows.

Year	Amount at beginning of the year	Interest receivable	Withdrawals (Rs.)	Amount at the end of the year
1	800,000	80,000	-	880,000
2	880,000	88,000	(500,000)	468,000
3	468,000	46,800	(400,000)	114,800
4	114,800	11,480	-	126,280

Exercise 01

A company has an investment of Rs. 300 million. This investment is expected to earn interest of 10% two more years, and 6% for 3 years after that. The company expects to withdraw Rs. 40 million from the investment at the end of each year beginning at the end of year 1. Calculate the amount that will be in the investment fund at the end of 5 years.

Amortization Schedule

An Amortization Schedule is a statement which shows the outstanding amount of a loan period by period.

Worked Example

Example ①

A customer of your firm has purchased a computer costing Rs. 160,000. The customer has paid a Rs. 60,000 deposit and has agreed to pay off the purchase price by installments of Rs. 35,000 per year, payable at the end of each year. Interest is charged on the outstanding balance at 17% per year.

(a) Draw up a schedule of payments until the debt is paid off.
(b) State the number of full payments of Rs. 35,000 is made.
(c) State the value of final payment.
(d) Calculate the amount paid in total for the computer.

Solution

Year	Amount outstanding at beginning	Interest payable	Re payment	Amount outstanding at the end
1	100,000	17,000	(35,000)	82,000
2	82,000	13,940	(35,000)	60,940
3	60,940	10,360	(35,000)	36,300
4	36,300	6,171	(35,000)	7,471
5	7,471	1,270	(8,741)	0

(a) 4 Full payments.
(b) Final payment Rs. 8,741
(c) Total amount to be paid = 60,000 + (4 × 35,000) + 8,741
= Rs. 208,741

FINANCIAL MATHEMATICS - II

* Present Value of Money
* Annuities / Perpetuity
* Basic Project Evaluation

PRESENT VALUE OF MONEY

Present value describes how much a future sum of money worth today. Sometimes it can be described as discounting. Formula for discounting is given as follows.

$$X = S \times DCF$$

Where $DCF = (1+r)^{-n}$

* S = Sum of money received after n time periods (Future value of money)
* X = Present value of that sum (Present value of money)
* r = Rate of Return
* n = No. of Time Periods
* DCF = Discounting Factor

Worked Examples

Example ①

What is the present value of Rs. 25,000 at the end of 4 years, if interest rate is 12% per annum compounded,

(a) Annually
(b) Semi - Annually
(c) Quarterly

Solution

(a) Annually

$X = S \times DCF$
S = Rs. 25,000
r = 0.12 (annually)
n = 4 (years)

$X = 25{,}000 \times 0.636$
X = Rs. 15,900

From a Present value table

	12%
4	0.636

$DCF = (1.12)^{-4} = 0.636$

(b) Semi - Annually

X = S × DCF
S = Rs. 25,000
r = 0.12/2 = 0.06 (Semi-annually)
2n = 4×2 = 8 (Semi-annual period)

X = 25,000 × 0.627
X = Rs. 15,675

From a Present value table

	6%
8	0.627

DCF = $(1.06)^{-8}$ = 0.627

(c) Quarterly

X = S × DCF
S = Rs. 25,000
r = 0.12/4 = 0.03 (Quarterly)
2n = 4×4 = 16 (No. of quarterly)

X = 25,000 × 0.623
X = Rs. 15,575

From a Present value table

	3%
16	0.623

DCF = $(1.06)^{-16}$ = 0.623

Exercise 01

Relate the investments on the left side to the corresponding net present values on right side,

Investment		Present Value
(a)	Rs. 20 million in 4 years of Discount Rate of 5%	19.5 Rs. Mn.
(b)	Rs. 30 million in 5 years of Discount Rate of 9%	28.95 Rs. Mn.
(c)	Rs. 50 million in 3 years of Discount Rate of 20%	16.46 Rs. Mn.

ANNUITIES

An annuity an agreement where by a person pays or receives a fixed amount at the end or beginning of each period. It is a sequence of fixed equal payments or receipts made over uniform time intervals without any interruption.

Examples

* Weekly wages
* Monthly salaries
* Pension scheme
* Insurance Premiums
* Housing Loan Repayments
* Hire-purchase Payments

Annuities may be paid

* At the end of payment intervals. (Ordinary annuity)
* At the beginning of payment intervals. (A due annuity)

Present Value of an Annuity

Present Value of an Annuity is sum of all present values of payments made at different times. If can be evaluate by using following formulae.

Present Value of an Ordinary Annuity

$$PV = \text{Installment} \times \text{Cumulative DCF}_{r\%,\ n\ \text{years}}$$

Where,

$$\text{Cumulative DCF} = \frac{1-(1+r)^{-n}}{r}$$

* r = Rate of Discount
* n = Life of an annuity

Present Value of an Annuity Due

PV = Installment (1 + Cumulative DCF)
r%, (n-1) years

Where,

Cumulative DCF = $\frac{1-(1+r)^{-n}}{r}$

* r = Rate of Discount
* n = Life time of an annuity

Mortgages

It is a type of a loan that is secured with real estate or property.

Perpetuity

Perpetuity is some as an annuity that payments carry on infinitely. It can be given as follows.

$PV = \frac{A}{r}$

* A = Size of installment
* r = Cost of capital / Rate of Discount

Worked Examples

Example ①

A farmer is to lease a field for 6 years at an annual rent of Rs. 50,000, the rentals being paid at the beginning of each year, calculate the present value of lease at 7%.

Solution

PV = Installment (1 + Cumulative DCF)
Installment = Rs. 50,000
r = 0.07
n-1 = 6-1 = 5
PV = 50,000 (1+4.100)
PV = 50,000 × 5.100
PV = Rs. 255,000

From a Cumulative PV table

	7%
5	4.100

Example ②

An individual has taken out a mortgage of Rs. 1,500,000 at a fixed interest rate of 5% per annum over 20 years. Repayments will be one year after the mortgage is taken out. Evaluate the annual repayment.

Solution

PV = Installment × Cumulative DCF)
PV = Rs. 1,500,000
r = 0.05
n = 20

1,500,000 = Installment (12.462)

$\frac{1{,}500{,}000}{12.462}$ = Installment

Rs. 120,366 = Installment

From a Cumulative PV table

	5%
20	12.462

Example ③

A client of yours has asked for your advised to chose one of the two contracts awarded to him. The first contract generates net cash inflows of an initial Rs. 10 million at the start of contract and the 1 million at end of the 4 years of the contract. The second contract also lasts for 4 years and generates net cash inflows of Rs. 4 million at the end of each year. The client's current rate of interest is 8% per annum. Explain with financial calculations the contract that you would advise, the company to choose?

Solution

Contract I

PV = Installment × Cumulative DCF)
Installment = Rs. 1,000,000
r = 0.08
n = 4

PV = 1,000,000 × 3.312
PV = 3,312,000
Total PV = 10,000,000 + 3,312,000
= Rs. 13,312,000

From a Cumulative PV table

	8%
4	3.312

Contract II

PV = Installment × Cumulative DCF)
Installment = Rs. 4,000,000
r = 0.08
n = 4

PV = 4,000,000 × 3.312
PV = Rs. 13,248,000

From a Cumulative PV table

	8%
4	3.312

Contract I is more profitable hence it is recommended.

Example ④

The annual year end income of Rs. 20,000 is required in perpetuity. There is a fixed interest rate of 8% each year and administrative charges could be ignored. Calculate the lump sum investment necessary now in order to obtain the perpetuity.

Solution

$PV = \frac{A}{r}$

$A = Rs.\ 20,000$

$r = 0.08$

$PV = \frac{20,000}{0.08}$

$PV = Rs.\ 250,000$

Exercise 01

A fixed interest Rs. 400,000 mortgage, with annual interest compounded at 6% each year, is to be repaid by 15 equal year end repayments of Rs. 'A'. Evaluate value of 'A'.

Exercise 02

The annual rent of a building is Rs. 120,000, payable in advance at the beginning of each year. At an interest rate in 14%, the present value of the rental payments is Rs. 531,960 compute the length of lease.

Exercise 03

ABC Ltd. wants to undertake a project which costs Rs. 2 million now and generates an annual cash flow of Rs. 250,000 per annum for every year in perpetuity. Discuss whether project is viable if rate of Discount is 12%.

Amortization Schedule

An Amortization Schedule is a specification, period by period of the state of the debt.

Worked Examples

Example ①

A mortgages of Rs. 1,000,000 is arrange now for 5 years at a rate of interest of 11%. Interest is compounded on the balance outstanding at the end of each year. The loan is repaid by 5 annual installments, the first being due after the end of one complete year.

(a) Calculate gross annual installments.

(b) State the amount outstanding after two complete years by preparing an amortization schedule.
The rate of interest charges to 13% after two complete years.

(c) Calculate the value of revised annual installment.

Solution

(a) Gross Annual Installments

PV = Installment × Cumulative DCF)
PV = Rs. 1,000,000
r = 0.11
n = 5

From a Cumulative PV table

	11%
5	3.696

1,000,000 = Installment (3.696)

$\frac{1,000,000}{3.696}$ = Installment

Rs. 270,563 = Installment

(b) Amortization Schedule

Year	Amount outstanding at beginning	Interest payable	Re payment	Amount outstanding at the end
1	1,000,000	110,000	270,563	839,437
2	839,437	92,338	270,563	661,212

(c) Value of Revised Annual Installment

PV = Installment × Cumulative DCF)
PV = Rs. 661,212
r = 0.13
n = 3

From a Cumulative PV table

	13%
3	2.361

661,212 = Installment (2.361)

$\frac{661,212}{2.361}$ = Installment

Rs. 280,056 = Installment

Exercise 04

A company obtains a loan of Rs. 50,000 at 6% interest per annum repayable in equal installments at every year and over the next 5 years. Calculate the annual payment necessary to amortize the debt and prepare amortization table.

BASIC PROJECT EVALUATION

A project should be evaluated before it is undertaken. That is the expenses to be incurred by the project and the return from the project are studied carefully and then whether the project makes a profit or not is analysed. This analysis is known as project evaluation.

The expenses incurred for the project at various points of time are known as cash outflows and the return from the project when considered in terms of cash are known as cash inflows.

Net Present Value Method.

When we subtract total present value of all cash outflows from total present value of all cash inflows, then the result obtained is known as net present value, which is denoted by NPV.

* If NPV of a project is positive, the project is in profits and hence it is considered to be acceptable.
* If NPV of a project is negative, the project makes a loss and hence it is considered to be an acceptable.
* In NPV of a project is zero, the position is known as break even situation.

Worked Examples

Example ①

A company purchased a machine now for Rs. 1,000,000. The accountant of the firm estimates that the machine would contribute Rs. 250,000 per annum to profits for next 5 years, after which point of time it can disposed for Rs. 50,000. Calculate NPV of machine if the rate of discount is 5% per annum.

Solution

Prepare a table as follows.

Year	Cash Flow (Rs.)	Discount Factor at 5%	Present Value
0	(1,000,000)	1.000	(1,000,000)
1	250,000	0.952	238,000
2	250,000	0.907	226,750
3	250,000	0.864	216,000
4	250,000	0.823	205,750
5	250,000+50,000	0.784	235,200
		NPV ⟶	221,700

Example ②

Star Ltd. has developed a vehicle security and is considering manufacturing and marketing a new product.

The following estimates of costs and revenue for the product over the 4 years have been made.

Sales Forecast.

Year	Estimated Sales (units)	Selling Price (per unit)
1	5,000	2,500
2	15,000	2,300
3	22,000	2,000
4	15,000	2,000

New plant and machinery will be purchased at a cost of Rs. 20 million, this will have a re-sale value of 2 million at the end of 4 years.

Material and labour costs will be Rs. 1,000 per units in year 1, rising by Rs. 20 per unit in each succeeding year.

Fixed overheads attributable to the product will be Rs. 2 million for the first two years rising by 10% in year 3 and by a further 5% in year 4.

The cost of capital to the company is 12%.

Calculate the net present value of the project and comment on whether the investment should be initialed?

Solution

Cash inflows for the project can be estimated as follows.

Year	Revenue	Material and labour cost	Fixed cost	Net cash inflow
1	12,500,000	(5,000,000)	(2,000,000)	5,500,000
2	34,500,000	(15,300,000)	(2,000,000)	17,200,000
3	44,000,000	(22,880,000)	(2,200,000)	18,920,000
4	30,000,000	(15,900,000)	(2,310,000)	11,790,000

NPV of project

Year	Cash Flow (Rs.)	Discount Factor at %	Present Value
0	(20,000,000)	1.000	(20,000,000)
1	5,500,000	0.893	4,119,500
2	17,200,000	0.797	13,708,400
3	18,920,000	0.712	13,471,040
4	13,790,000	0.636	8,770,440
		NPV ⟶	20,861,380

Project is profitable since NPV is positive.

Exercise 01

Two projects are expected to generate the following cash inflows during next 4 year period. Initial cost of each project is Rs. 215,000.

Assuming that these cash inflows occur at the end of each year, calculate NPV of each project. The cost of capital is 20% per annum.

Year	Project (A) Rs.	Project (B) Rs.
1	200,000	150,000
2	50,000	150,000
3	50,000	50,000
4	100,000	50,000

Internal Rate of Return

Internal rate of return is the rate of interest which Net Present Value becomes to zero. It is the rate at which a project makes neither profits nor losses. It can be evaluate by using following formula.

$$IRR = \frac{N_1 r_2 - N_2 r_1}{N_1 - N_2}$$

* IRR = Internal Rate of Return
* r_1 = Lowest Discount Rate
* r_2 = Highest Discount Rate
* N_1 = NPV at Lowest Discount Rate (r_1)
* N_2 = NPV of Highest Discount Rate (r_2)

Worked Examples

Example ①

An investment has a Net Present Value of 4000 at 10% and one of -2000 at 15%> Calculate approximate IRR.

Solution

$$IRR = \frac{N_1 r_2 - N_2 r_1}{N_1 - N_2}$$

$r_1 = 0.1$
$r_2 = 0.15$
$N_1 = +4000$
$N_2 = -2000$

$$IRR = \frac{(4000 \times 0.15) - (-2000 \times 0.1)}{4000 - (-2000)}$$

$$IRR = \frac{600 + 200}{6000}$$

IRR = 13.33%

Example ②

The following two capital investment, involve the purchase, use and final disposal of two machine m_1 and m_2. The life of two machines is 4 years at the end of which they can be sold for 10% of their purchase price. Assume that company's cost of capital be 10% per annum.

Year	Cash flow (Rs. million)	
	Machine (m_1)	Machine (m_2)
0	(50)	(40)
1	20	30
2	20	10
3	20	10
4	10	10

Select which investment is acceptable by using

(a) NPV method
(b) IRR method

Solution

(a) By using NPV method

Prepare a table as follows.

For machine m_1

Year	Cash Flow (Rs.)	Discount Factor at 10%	Present Value
0	(50)	1.000	(50)
1	20	0.909	18.18
2	20	0.826	16.52
3	20	0.751	15.02
4	10 + 5	0.683	10.25
			9.97

For machine m_2

Year	Cash Flow (Rs.)	Discount Factor at 10%	Present Value
0	(40)	1.000	(40)
1	30	0.909	27.27
2	10	0.826	8.26
3	10	0.751	7.51
4	10 + 4	0.683	9.56
			12.60

Machine m_2 is recommended as it makes higher profits.

(b) By using IRR method

For machine m_1

Year	Cash Flow (Rs.)	Discount Factor at 20%	Present Value
0	(50)	1.000	(50)
1	20	0.833	16.66
2	20	0.694	13.88
3	20	0.579	11.58
4	15	0.482	7.23
			(0.65)

$$IRR = \frac{N_1 r_2 - N_2 r_1}{N_1 - N_2}$$

$$IRR = \frac{(9.97 \times 20) - (-0.65 \times 10)}{9.97 - (-0.65)}$$

$$IRR = \frac{205.9}{10.62}$$

$$IRR = 19.39\%$$

For machine m_2

Year	Cash Flow (Rs.)	Discount Factor at %	Present Value
0	(40)	1.000	(40)
1	30	0.833	24.99
2	10	0.694	6.94
3	10	0.579	5.79
4	14	0.482	6.75
			4.47

$$\text{IRR} = \frac{N_1 r_2 - N_2 r_1}{N_1 - N_2}$$

$$\text{IRR} = \frac{(12.60 \times 20) - (4.47 \times 10)}{12.60 - 4.47}$$

$$\text{IRR} = \frac{207.3}{8.13}$$

$$\text{IRR} = 25.46\%$$

Machine m_2 is recommended as it more profitable.