

# Time Series Analysis

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## Introduction

A time series is a series of figure recorded over a time.

Examples

- (a) Output of a factory each day for last month.
- (b) Monthly sales over the last two years.
- (c) Total annual cost for last 10 years.
- (d) The retail price index each month for last 10 years.

Components of a Time series

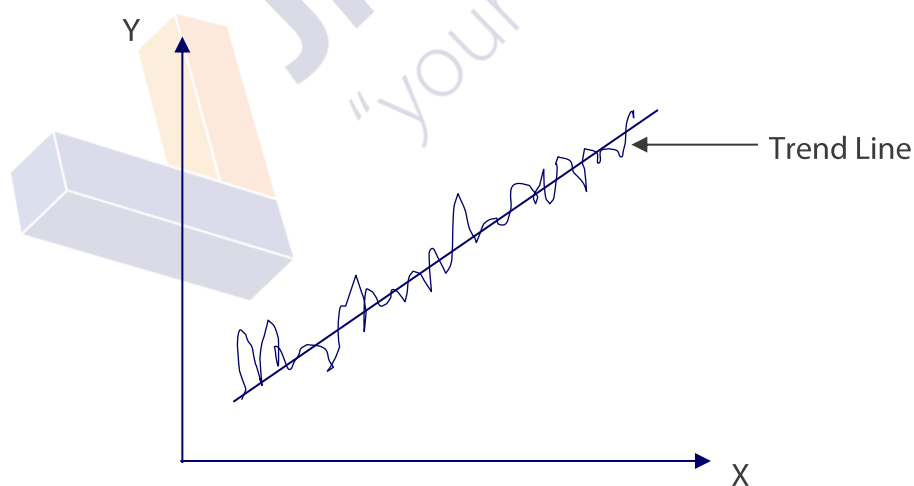
The main features of a time series is as follows.

- Basic trend
- Seasonal variations
- Cyclical variations
- Random or irregular variations

### Basic Trend

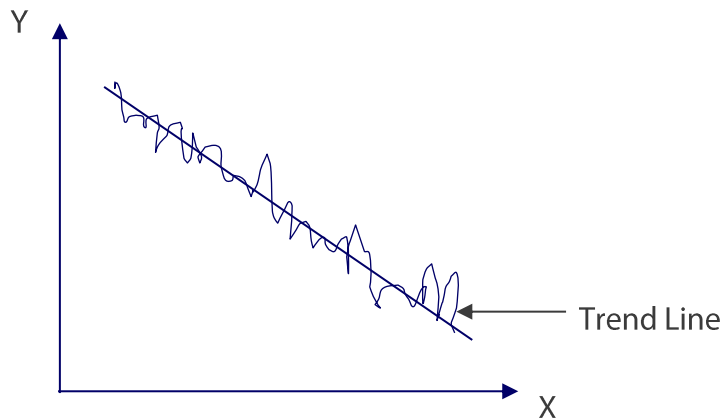
The basic trend refers to the general direction in which the graph of the time series appears to be going over long interval of time. This movement can be represented on the graph by a straight line or curve.

Consider the following examples.



Time series has an upward trend

## Time Series Analysis



Time series has a downward trend



Time series has a uniform trend

## Seasonal Variations

Seasonal variations are almost identical patterns which a time series follows during corresponding intervals of successive periods. In general seasonal movements refer to a period of one year, periods of months, weeks, days and even hours may be considered depending on type of data available.

Examples:

- Shops might expect higher sales shortly before Christmas and new year.
- Sale of ice-cream will be higher in summer than the winter.
- Sales must be higher on Friday and Saturday than on Monday.
- The telephone network may heavily used at particular time of day.

## Cyclical Variations

Cyclical variations refer to long term oscillations or swings about the trend line or curve. They may not follow similar patterns after equal intervals of time. For cyclical variations to be appeared, data must be available for very long periods of time.

Example:

Trade cycle representing intervals of prosperity and depression

## Random or Irregular Variations

These are the odd movements of time series curve, which fit into no pattern at all.

Example:

Production of a certain factory goes down due to strike of workers

## Summersing Components of a Time series

The component of a time series can be summarized by using following equations.

### Additive Model

$$Y = T + S + C + I$$

### Multiplicative Model

$$Y = T \times S \times C \times I$$

- Y = Actual Time series  
T = Basic Trend  
S = seasonal Variations  
C = Cyclical Variations  
I = Random or Irregular Variations

### Estimation of Basic Trend

Following methods can be used to estimating a basic trend of time series.

- (i) Free hand method
- (ii) Method of moving averages
- (iii) Method of least squares

### Free Hand Method

We can plot the points with normal scales on XY coordinate axes and get the scatter diagram. By looking into the pattern of points on the scatter diagram we can draw the best fitting line to pass through the points. Since this line is drawn according to an individual's way of thinking, different people will get different lines, for the same set of data. Therefore this is not so reliable to use for predictions.

## Method of Moving Averages

Moving averages is an analytical technique that uses to separate out seasonal factors as well as basic trend.

### Moving Average of an odd Number of Result

#### Worked Example

Following table gives the sales from 2000 – 2006. Required to take a moving average at annual sales over a period of 3 years.

Year	Sales Rs' '000 (Y)	Moving Total (3)	Moving Average (3) (T)
2000	390	-	-
2001	380	1,230	410
2002	460	1,290	430
2003	450	1,380	460
2004	470	1,360	453
2005	440	1,410	470
2006	500	-	-

Note:

- The moving average series has 5 figures relating to the years 2001 to 2005.
- There is an upward trend in sales which is more noticeable from series of moving averages than the original series.

#### Example (1)

Following table gives the production from Janu – July in year 2010. Required to take a moving average at monthly production over a period of 3 months.

Month	Jan.	Feb.	March	April	May	June	July
Production 000' (Units)	40	60	20	70	90	80	40

### Moving Average of an Even Number of Result

In the previous example moving averages were taken of the result in odd number of time periods, and the average then related to the mid-point of overall period.

If moving averages were taken of result in an even number of time periods the basic technique would be same, but the mid-point of overall period would not relate to a single period. therefore centered average should be calculate.

**Worked Example**

The sales manager of a company has collected following data regarding average monthly sales revenue during the years 1995 - 2005.

Year	Sales Revenue (Rs. '000) (Y)	Moving Total (4)	Moving Average (4)	Centered Average (T)
1995	113			
1996	93			
		421	105.25	
1997	106			103.375
		406	101.50	
1998	109			101.875
		409	102.25	
1999	98			99.625
		388	97.00	
2000	96			95.500
		376	94.00	
2001	85			94.750
		382	95.5	
2002	97			96.25
		388	97.00	
2003	104			97.25
		390	97.50	
2004	102			
2005	87			

Note:

There is a downward trend which is more noticeable than the original series.

**Example (2)**

Following table gives the quarterly sales from 2005 to 2007. Required to take moving average of quarterly sales.

Year	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>
2005	600	840	420	720
2006	640	860	420	740
2007	670	900	430	760

### Method of Least Squares

If after plotting a time series on a graph the trend appears to be approximately linear, it can be estimated by using method of least square. Time is always the independent variable, in these cases we should be plotted along the X-axis.

When dealing with time series where X values are in successive years or months or days. In this situation we can take the X variable which measure the time to be zero for the middle period.

Let  $X = 0$  for the year 2002. Then we get X values as 1, 2, 3, 4 ..... for the years greater than 2002 and similarly we get X values as -1, -2, -3, -4 ..... for the years less than 2002.

### Worked Example

The following time series shows the total annual net sales of an electric company for the years 1998 - 2006.

Year	1998	1999	2000	2001	2002	2003	2004	2005	2006
Sales Rs. Mn.	145	158	164	144	152	201	190	193	196

- (i) Evaluate equation of a trend line
- (ii) Forecast the sales in year 2007



**ANSWER**

Prepare a table as follows.

Year	X	Sales (Y)	XY	X <sup>2</sup>
1998	-4	145	-580	16
1999	-3	158	-474	09
2000	-2	164	-328	04
2001	-1	144	-144	01
2002	0	152	0	00
2003	1	201	201	01
2004	2	190	380	04
2005	3	193	579	09
2006	4	196	784	16
	$\sum x = 0$	$\sum y = 1543$	$\sum xy = 418$	$\sum x^2 = 60$

Equation of a trend line  $Y = bx + a$

$$b = \frac{n \sum y - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$b = \frac{9 \times 418 - (0)(1543)}{9 \times 60 - (0)^2}$$

$$b = \frac{418}{60}$$

$$\underline{b = 6.97}$$

$$a = \frac{\sum y}{n} - \frac{b \sum x}{n}$$

$$a = \frac{1543}{9} - \frac{6.97 \times 0}{9}$$

$$a = \frac{1543}{9}$$

$$\underline{a = 171.44}$$

Trend line equation ( $Y = bx + a$ )

$$Y = 6.97x + 171.44$$

When  $x = 5$  (2007)

$$Y = 6.97(5) + 171.44$$

$$Y = 206.3 \text{ Rs. Mn}$$

**Example (1)**

Following table relates to the population in thousands of a particular town in between 2000 – 2008

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
Population 000's	25	31	36	44	48	53	60	62	67



**Example (2)**

The level of working capital required by a small company for last six years is given below.

Year	2001	2002	2003	2004	2005	2006
Working Capital Rs. (lakhs)	10	12	13	13	14	15

- (i) Calculate equation of a trend line.
- (ii) Estimate the level of working capital required during the year 2007.

**Finding Seasonal Variations**

Additive model for time series is  $Y = T + S + C + I$  we assume that I and C are relatively small,

$$Y = T + S$$

$$Y - T = S$$

Multiplicative model for time series is  $Y = T \times S \times C \times I$  we assume that I and C are relatively small,

$$Y = T \times S$$

$$\frac{Y}{T} \times 100 = S$$

**Worked examples**

Output of a factory vary with days of week. Output of last 3 weeks as given below.

Week	Mon.	Tue.	Wed.	Thu.	Fri.
1	80	104	94	120	62
2	82	110	97	125	64
3	84	116	100	130	66

Calculate seasonal variations for each day by assuming an additive model.

Week	Day	Actual Figure (Y)	Moving Total (S)	Moving Average (T)	Seasonal Variations $S = Y - T$
01	Mon.	80	-	-	-
	Tue.	104	-	-	-
	Wed.	94	460	92.0	+2.0
	Thu.	120	462	92.4	+27.6
	Fri.	62	468	93.6	-31.6
02	Mon.	82	471	94.2	-12.2
	Tue.	110	476	95.2	+14.8
	Wed.	97	478	95.6	+1.4
	Thu.	125	480	96.0	+29.0
	Fri.	64	486	97.2	-33.2
03	Mon.	84	489	97.8	-13.8
	Tue.	116	494	98.8	+17.2
	Wed.	100	496	99.2	+0.8
	Thu.	130	-	-	-
	Fri.	06	-	-	-

Prepare a table as follows.

Week	Mon.	Tue.	Wed.	Thu.	Fri.
01	-	-	+2.0	+27.6	-31.6
02	-12.2	+14.	+1.4	+29.0	-33.2
03	-13.8	+17.2	+0.8	-	-
<b>Average unadjusted</b>	-13.0	+16.0	+1.4	+28.3	-32.4

Sum of the averages for 5 days = +0.3

Sum of the averages for 5 days should be = 0

To adjust the mean values we subtract = +0.06 from un-adjusted mean.

Monday =  $-13.0 - 0.06 = -13.06$

Tuesday =  $+16.0 - 0.06 = +15.94$

Wednesday =  $+1.4 - 0.06 = +1.34$

Thursday =  $+28.3 - 0.06 = +28.24$

Friday =  $-32.4 - 0.06 = -32.46$

**Example (2)**

By using the data given below. Calculate seasonal variations for each quarter.

Year	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>
2010	200	120	160	280
2011	220	140	140	300
2012	200	120	180	320

Year (Quarter)	Actual Figure (Y)	Moving Total (4)	Moving Average	Centered Average (T)	Seasonal Variations $s = \frac{Y}{T} \times 100$
2007					
Q <sub>1</sub>	200				-
Q <sub>2</sub>	120				-
		760	190		
Q <sub>3</sub>	160			192.5	83.12
		780	195		
Q <sub>4</sub>	280			197.5	141.77
		800	200		
2008					
Q <sub>1</sub>	220			197.5	111.39
		780	195		
Q <sub>2</sub>	140			197.5	70.89
		800	200		
Q <sub>3</sub>	140			197.5	70.89
		780	195		
Q <sub>4</sub>	300			192.5	155.84
		760	190		
2008					
Q <sub>1</sub>	200			195	102.56
		800	200		
Q <sub>2</sub>	120			202.5	59.26
		820	205		
Q <sub>3</sub>	180				-
Q <sub>4</sub>	320				-

Prepare a table as follows.

Year	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>
2007	-	-	83.12	141.77
2008	111.39	70.89	70.89	155.84
2009	102.56	59.26	-	-
Average (Un-adjusted)	106.98	65.07	77.00	148.81

Index for each quarter is equal to 100. The sum of the means should add up to 400.

Sum of un-adjusted means = 397.86

Adjusted means

$$Q_1 = \frac{106.98}{397.86} \times 400 = \underline{107.56}$$

$$Q_2 = \frac{65.07}{397.86} \times 400 = \underline{65.42}$$

$$Q_3 = \frac{77.00}{397.86} \times 400 = \underline{77.41}$$

$$Q_4 = \frac{148.81}{397.86} \times 400 = \underline{149.61}$$

### De- Time series

Seasonal indices help us to remove the effects of seasonal variation on the time series. The use of indices to remove seasonal variations is known as de-seasonalizing data.

When using the additive model de-seasonalized data can be obtained by subtracting the seasonal fluctuations from the original data.

When using the multiplicative model de-seasonalized data can be obtained by, original data dividing by seasonal fluctuations.

#### Additive Model

De – senasonalized

$$\text{Data} = Y - S$$

#### Multiplicative Model

De – senasonalized

$$\text{Data} = \frac{Y}{S \times 100}$$