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# Time Series Analysis

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## Introduction

A time series is a series of figure recorded over a time.

### Examples

(a) Output of a factory each day for last month.
(b) Monthly sales over the last two years.
(c) Total annual cost for last 10 years.
(d) The retail price index each month for last 10 years.

## Components of a Time Series

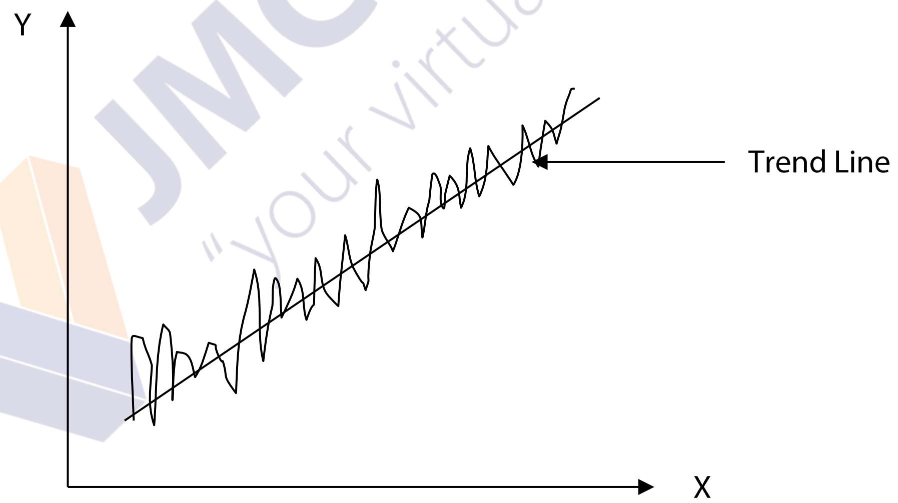
The main features of a time series is as follows.

- Basic trend
- Seasonal variations
- Cyclical variations
- Random or irregular variations

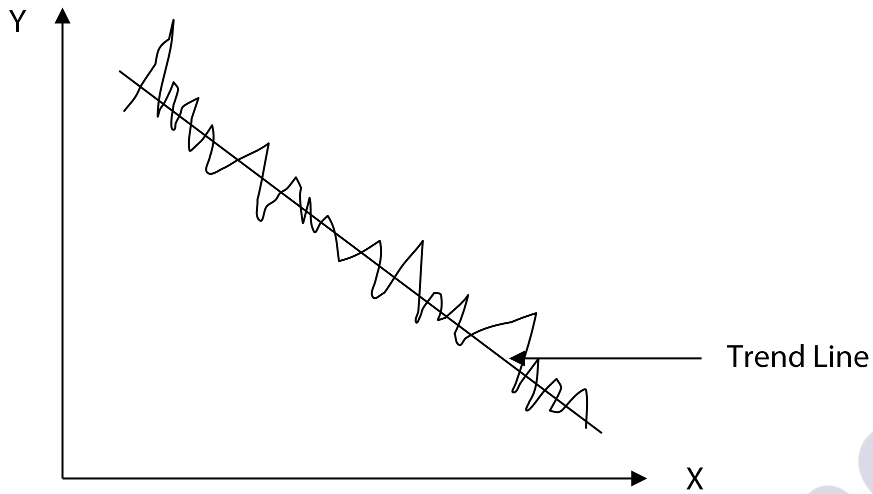
## Basic Trend

The basic trend refers to the general direction in which the graph of the time series appears to be going over long interval of time. This movement can be represented on the graph by a straight line or curve.

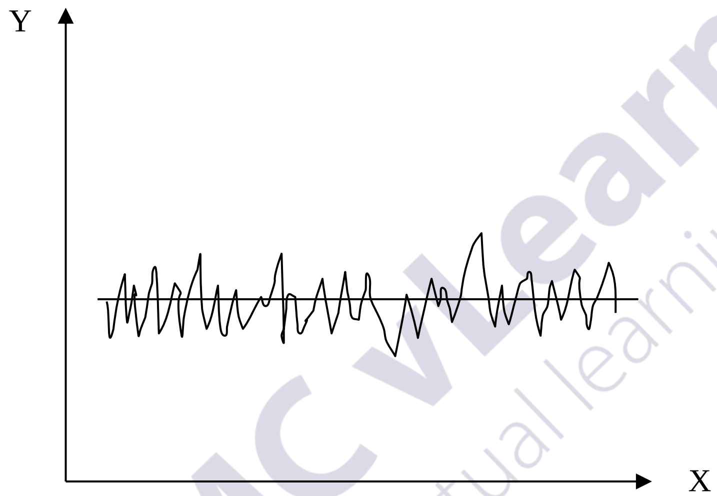
Consider the following examples.




Time series has an upward trend




Time series has a downward trend




Time series has a uniform trend

## Seasonal Variations

Seasonal variations are almost identical patterns which a time series follows during corresponding intervals of successive periods. In general seasonal movements refer to a period of one year, periods of months, weeks, days and even hours may be considered depending on type of data available.

**Examples:**

- Shops might expect higher sales shortly before Christmas and New Year.
- Sale of ice-cream will be higher in summer than the winter.
- Sales must be higher on Friday and Saturday than on Monday.
- The telephone network may heavily use at particular time of day.

## Cyclical Variations

Cyclical variations refer to long term oscillations or swings about the trend line or curve. They may not follow similar patterns after equal intervals of time. For cyclical variations to be appeared, data must be available for very long periods of time.

### Example:

- Trade cycle representing intervals of prosperity and depression

## Random or Irregular Variations

These are the odd movements of time series curve, which fit into no pattern at all.

### Example:

- Production of a certain factory goes down due to strike of workers

## Summarizing Components of A Time Series

The component of a time series can be summarized by using following equations.

### Additive Model

$$Y = T + S + C + I$$

### Multiplicative Model

$$Y = T \times S \times C \times I$$

| | | |
|---|---|---|
| Y | = | Actual Time Series |
| T | = | Basic Trend |
| S | = | seasonal Variations |
| C | = | Cyclical Variations |
| I | = | Random or Irregular Variations |

### Estimation of Basic Trend

Following methods can be used to estimating a basic trend of time series.

(i) Free hand method

(ii) Method of moving averages

(iii) Method of least squares

### Free Hand Method

We can plot the points with normal scales on XY coordinate axes and get the scatter diagram. By looking into the pattern of points on the scatter diagram we can draw the best fitting line to pass through the points. Since this line is drawn according to an individual's way of thinking, different people will get different lines, for the same set of data. Therefore this is not so reliable to use for predictions.

### Method of Moving Averages

Moving averages is an analytical technique that uses to separate out seasonal factors as well as basic trend.

### Moving Average of an odd Number of Result

**Worked Example**

Following table gives the sales from 2000 – 2006. Required to take a moving average at annual sales over a period of 3 years.

| Year | Sales Rs' '000 (Y) | Moving Total (3) | Moving Average (3) (T) |
|---|---|---|---|
| **2000** | 390 | - | - |
| **2001** | 380 | 1,230 | 410 |
| **2002** | 460 | 1,290 | 430 |
| **2003** | 450 | 1,380 | 460 |
| **2004** | 470 | 1,360 | 453 |
| **2005** | 440 | 1,410 | 470 |
| **2006** | 500 | - | - |

Note:

- The moving average series has 5 figures relating to the years 2001 to 2005.
- There is an aupord trend in sales which is more noticeable from series of moving averages than the original series.

### Example 1

Following table gives the production from Janu – July in year 2010. Required to take a moving average at monthly production over a period of 3 months.

| Month | Jan. | Feb. | March | April | May | June | July |
|---|---|---|---|---|---|---|---|
| **Production 000' (Units)** | 40 | 60 | 20 | 70 | 90 | 80 | 40 |

### Moving Average of an Even Number of Result

In the previous example moving averages were taken of the result in odd number of time periods, and the average then related to the mid-point of overall period.

If moving averages were taken of result in an even number of time periods the basic technique would be same, but the mid-point of overall period would not relate to a single period. Therefore centered average should be calculate.

### Worked Example

The sales manager of a company has collected following data regarding average monthly sales revenue during the years 1995 – 2005.

| Year | Sales Revenue (Rs. '000) (Y) | Moving Total (4) | Moving Average (4) | Centered Average (T) |
|---|---|---|---|---|
| **1995** | 113 | | | |
| **1996** | 93 | | | |
| | | 421 | 105.25 | |
| **1997** | 106 | | | 103.375 |
| | | 406 | 101.50 | |

| | | | | |
|---|---|---|---|---|
| **1998** | 109 | | | 101.875 |
| | | 409 | 102.25 | |
| **1999** | 98 | | | 99.625 |
| | | 388 | 97.00 | |
| **2000** | 96 | | | 95.500 |
| | | 376 | 94.00 | |
| **2001** | 85 | | | 94.750 |
| | | 382 | 95.5 | |
| **2002** | 97 | | | 96.25 |
| | | 388 | 97.00 | |
| **2003** | 104 | | | 97.25 |
| | | 390 | 97.50 | |
| **2004** | 102 | | | |
| **2005** | 87 | | | |

**Note:**

There is a downward trend which is more noticeable than the original series.

### Example 2

Following table gives the quarterly sales from 2005 to 2007. Required to take moving average of quarterly sales.

| Year | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ |
|---|---|---|---|---|
| **2005** | 600 | 840 | 420 | 720 |
| **2006** | 640 | 860 | 420 | 740 |
| **2007** | 670 | 900 | 430 | 760 |



## Method of Least Squares

If after plotting a time series on a graph the trend appears to be approximately linear, it can be estimated by using method of least square. Time is always the independent variable, in these cases we should be potted along the X-axis.

When dealing with time series where X values are in successive years or months or days. In this situation we can take the X variable which measure the time to be zero for the middle period.

Let X = 0 for the year 2002. Then we get X values as 1, 2, 3, 4 …… for the years greater than 2002 and similarly we get X values as -1, -2, -3, -4 ….. For the years less than 2002.

**Worked Example**

The following tie series shows the total annual net sales of an electric company for the years 1998 - 2006.

| Year | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
|---|---|---|---|---|---|---|---|---|---|
| **Sales Rs. Mn.** | 145 | 158 | 164 | 144 | 152 | 201 | 190 | 193 | 196 |

(i) Evaluate equation of a trend line

(ii) Forecast the sales in year 2007

Answer

Prepare a table as follows.

| Year | X | Sales (Y) | XY | $X^2$ |
|---|---|---|---|---|
| **1998** | -4 | 145 | -580 | 16 |
| **1999** | -3 | 158 | -474 | 09 |
| **2000** | -2 | 164 | -328 | 04 |
| **2001** | -1 | 144 | -144 | 01 |
| **2002** | 0 | 152 | 0 | 00 |
| **2003** | 1 | 201 | 201 | 01 |
| **2004** | 2 | 190 | 380 | 04 |
| **2005** | 3 | 193 | 579 | 09 |
| **2006** | 4 | 196 | 784 | 16 |
| | $\sum x = 0$ | $\sum y = 1543$ | $\sum xy = 418$ | $\sum x^2 = 60$ |

**Equation of a trend line** $(\boldsymbol{y} = \boldsymbol{bx} + \boldsymbol{a})$

$b = \frac{n\sum y-(\sum x)(\sum y)}{n\sum x^2 -(\sum x)^2}$

$b = \frac{9\times418-(0)(1.543)}{9\times60 -(0)^2}$

$b = \frac{418}{60}$

$b = 6.97$

$a = \frac{\sum y}{n} - \frac{b\sum x}{n}$

$a = \frac{1,543}{9} - \frac{6.97\times0}{9}$

$a = \frac{1,543}{9}$

$a = 171.44$

**Trend line equation** $(\boldsymbol{y} = \boldsymbol{bx} + \boldsymbol{a})$

Y = $6.97x + 171.44$

When x = 5 (2007)

Y = 6.97 (5) + 171.44

Y = 206.3 Rs. Mn

## Example 1

Following table relates to the population in thousands of a particular town in between 2000 – 2008

| Year | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
|---|---|---|---|---|---|---|---|---|---|
| Population 000's | 25 | 31 | 36 | 44 | 48 | 53 | 60 | 62 | 67 |

## Example 2

The level of working capital required by a small company for last six years is given below.

| Year | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
|---|---|---|---|---|---|---|
| Working Capital Rs. (lakhs) | 10 | 12 | 13 | 13 | 14 | 15 |

(i) Calculate equation of a trend line.

(ii) Estimate the level of working capital required during the year 2007.

## Finding Seasonal Variations

Additive model for time series is Y = T + S + C + I we assume that I and C are relatively small,

$$Y = T + S$$

$$Y - T = S$$

Multiplicative model for time series is Y = T × S × C × I we assume that I and C are relatively small,

$$Y = T \times S$$

$$\frac{Y}{T} \times 100 = S$$

## Worked examples

Output of a factory vary with days of week. Output of last 3 weeks as given below.

| Week | Mon. | Tue. | Wed. | Thu. | Fri. |
|---|---|---|---|---|---|
| 1 | 80 | 104 | 94 | 120 | 62 |
| 2 | 82 | 110 | 97 | 125 | 64 |
| 3 | 84 | 116 | 100 | 130 | 66 |

Calculate seasonal variations for each day by assuming an additive model

| Week | Day | Actual Figure (Y) | Moving Total (S) | Moving Average (T) | Seasonal Variations S = Y – T |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| | Mon. | 80 | - | - | - |
| | Tue. | 104 | - | - | - |
| **01** | Wed. | 94 | 460 | 92.0 | +2.0 |
| | Thu. | 120 | 462 | 92.4 | +27.6 |
| | Fri. | 62 | 468 | 93.6 | -31.6 |
| | Mon. | 82 | 471 | 94.2 | -12.2 |
| | Tue. | 110 | 476 | 95.2 | +14.8 |
| **02** | Wed. | 97 | 478 | 95.6 | +1.4 |
| | Thu. | 125 | 480 | 96.0 | +29.0 |
| | Fri. | 64 | 486 | 97.2 | -33.2 |
| | Mon. | 84 | 489 | 97.8 | -13.8 |
| | Tue. | 116 | 494 | 98.8 | +17.2 |
| **03** | Wed. | 100 | 496 | 99.2 | +0.8 |
| | Thu. | 130 | - | - | - |
| | Fri. | 06 | - | - | - |

Prepare a table as follows.

| Week | Mon. | Tue. | Wed. | Thu. | Fri. |
|---|---|---|---|---|---|
| **01** | - | - | +2.0 | +27.6 | -31.6 |
| **02** | -12.2 | +14. | +1.4 | +29.0 | -33.2 |
| **03** | -13.8 | +17.2 | +0.8 | - | - |
| **Average unadjusted** | -13.0 | +16.0 | +1.4 | +28.3 | -32.4 |

Sum of the averages for 5 days = +0.3

Sum of the averages for 5 days should be = 0

To adjust the mean values we subtract $\frac{0.3}{5}$ = +0.06 from un-adjusted mean.

Monday = -13.0   0.06 = -13.06

Tuesday = +16.0 – 0.06 = +15.94

Wednesday = +1.4 – 0.06 = +1.34

Thursday = +28.3 – 0.06 = -32.46

Friday = -32.4 – 0.06 = -32.46

## Example 2

By using the data given below. Calculate seasonal variations for each quarter.

| Year | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ |
|---|---|---|---|---|
| **2010** | 200 | 120 | 160 | 280 |
| **2011** | 220 | 140 | 140 | 300 |
| **2012** | 200 | 120 | 180 | 320 |

| Year (Quarter) | Actual Figure (Y) | Moving Total (4) | Moving Average | Centered Average (T) | Seasonal Variations $s\frac{y}{T}\times 100$ |
|---|---|---|---|---|---|
| **2007** $Q_1$ | 200 | | | | - |
| $Q_2$ | 120 | | | | - |
| | | 760 | 190 | | |
| $Q_3$ | 160 | | | 192.5 | 83.12 |
| | | 780 | 195 | | |
| $Q_4$ | 280 | | | 197.5 | 141.77 |
| | | 800 | 200 | | |
| **2008** $Q_1$ | 220 | | | 197.5 | 111.39 |
| | | 780 | 195 | | |
| $Q_2$ | 140 | | | 197.5 | 70.89 |
| | | 800 | 200 | | |
| $Q_3$ | 140 | | | 197.5 | 70.89 |
| | | 780 | 195 | | |
| $Q_4$ | 300 | | | 192.5 | 155.84 |
| | | 760 | 190 | | |
| **2008** $Q_1$ | 200 | | | 195 | 102.56 |
| | | 800 | 200 | | |
| $Q_2$ | 120 | | | 202.5 | 59.26 |
| | | 820 | 205 | | |
| $Q_3$ | 180 | | | | - |
| $Q_4$ | 320 | | | | - |

Prepare a table as follows.

| Year | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ |
|---|---|---|---|---|
| **2007** | - | - | 83.12 | 141.77 |
| **2008** | 111.39 | 70.89 | 70.89 | 155.84 |
| **2009** | 102.56 | 59.26 | - | - |
| **Average (Un-adjusted)** | 106.98 | 65.07 | 77.00 | 148.81 |

Index for each quarter is equal to 100. The sum of the means should add up to 400.

Sum of un-adjusted means = 397.86

Adjusted means

$Q_1 = \frac{106.98}{397.86} \times 400 = \underline{\underline{107.56}}$

$Q_2 = \frac{65.07}{397.86} \times 400 = \underline{\underline{65.42}}$

$Q_3 = \frac{77.00}{397.86} \times 400 = \underline{\underline{77.41}}$

$Q_4 = \frac{148.81}{397.86} \times 400 = \underline{\underline{149.61}}$

## De-seasonal zing Time Series

Seasonal indices help us to remove the effects of seasonal variation on the time series. The use of indices to remove seasonal variations is known as de-seasodling data.

When using the additive model de-seasonal zed data can be obtained by subtracting the seasonal fluctuations from the original data.

When using the multiplicative model de-seasonal zed data can be obtained by, original data dividing by seasonal fluctuations.

### Additive Model

**De – seasonal zed**

Data = Y – S

### Multiplicative Model

**De – seasonal zed**

Data = $\frac{Y}{S \times 100}$