

CHAPTER 14

(1) What is "Risk"?

Risk involves situations which may or may not occur, but their probability of occurrence can be calculated statistically and the frequency is predictable. In other words, risk relates to our expectations of what will happen. Risk arises because what actually happens could differ from what is intended or expected to happen.

(2) What is "Uncertainty"?

Uncertainty involves situations or events where outcome can not be predicted with the statistical confidence. The inability to predict the outcome is due to the lack of information.

Therefore, the risk is calculable whereas the uncertainty is subjective.

(3) Types of Risks

The risk is of paramount importance (most important than anything else) to organisations. Businesses must identify, evaluate, manage and report all the types of risks for their improved decision making. Risk can be classified in a number of ways.

(1) Business/Operational Risks:

The risks which are relating to the activities carried out by the entity. It may arise from the structure, systems, people, products or processes.

(2) Country risks:

This is the risk associated with undertaking transactions with, or holding assets in, a particular country.

Risk might be political, economic or from regulatory instability.

(3) Environmental risks:

These risks may occur due to the changes in political, economic, sociocultural, technological and legal environment.

(4) **Financial risks:**

This is relating to the financial operations of an entity. It includes four (04) types of risks.

- (i) **Credit risk:** Occurred due to the failure of another party to perform according to the terms of a contract.
- (ii) **Currency risk:** The risk of fluctuating the value of an instrument due to the changes in foreign exchange rates.
- (iii) **Interest rate risk:** This is the risk of affecting the financial comfortability of an entity due to the changes in interest rates.
- (iv) **Liquidity /Funding risk:** Risk of encountering financial difficulties in realizing assets or raising funds to meet financial commitments.

(5) **Reputational risks:**

This is the damage to an entity's reputation as a result of failure to manage other risks.

(6) **Strategic risks:**

These are the risks of deviating from the entity's strategies.

(4) Risk Management

Risk management is the process of understanding the risk and attempting to manage it in order to achieve its corporate objectives.

4.1 The concept of probability

Probability is a measure of likelihood of an event and it can be stated as a percentage (%), a ratio (x : y) or more usually as a number from 0 to 1. (zero to one).

Consider the following scenarios;

- If the probability = 0, it is impossible
- If the probability = 1, it is certain
- If the probability = $\frac{1}{2}$, a 50% chance of something happening
- If the probability = $\frac{1}{4}$, a 1 in 4 chance of something happening

4.1.1 How to assess the probability?

There are several ways of assessing the probability.

- (a) It is measurable with mathematical certainty.

Eg (1): If a coin is tossed, there is a 0.5 probability (chance) of getting the head and a 0.5 probability of getting the tail.

Eg (2): If a dice is thrown, there is a one-sixth (1/6) probability of getting six (6).

- (b) They may be measurable from an analysis of past experience.

- (c) Probability can be estimated from research or surveys.

$$\text{Probability (P)} = \frac{\text{Number of ways of achieving desired results}}{\text{Total number of possible outcomes}}$$

Eg: Suppose a box contain three red balls and two blue balls. If you close your eyes and pick a ball from box, calculate the probability that it will be a blue ball.

$$P(\text{blue}) = \frac{\text{Number of ways of achieving desired results}}{\text{Total number of possible outcomes}}$$

$$= \frac{2}{5} \quad \boxed{40\% \quad \text{or} \quad 0.4}$$

4.1.2 The simple addition law (OR Law)

The simple addition law for two mutually exclusive events, A and B, is as follows.

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

This is also known as "additive rule".

Eg: When you are throwing a dice, what is the probability of getting two (2) or six (6).

$$\text{Probability of getting 2} = 1/6$$

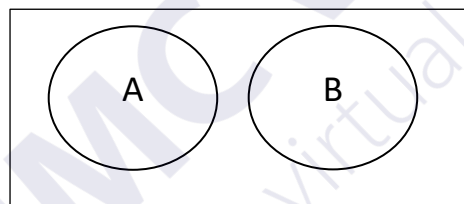
$$\text{Probability of getting 6} = 1/6$$

$$\begin{aligned} \therefore \text{Probability of getting 2 or 6} &= 1/6 + 1/6 \\ &= 2/6 = \underline{1/3} \end{aligned}$$

Mutually exclusive events are the events where the occurrence of one outcome excludes the possibility of any of the others happening.

In the above example, it is not possible to get both 2 and 6, at once. If you get 2, you will not get 6.

Venn diagram of mutually exclusive events:

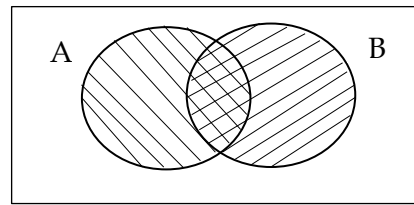


4.1.2.1 Additive rule for non-mutually exclusive events

The simple addition law for two non-mutually exclusive events, A and B, is as follows.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Venn diagram:



4.1.3 The simple multiplication law (AND Law)

The multiplication law for two independent events, A and B, is as follows.

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

❖ When A and B are mutually exclusive events,

Example:

A dice is thrown and a coin is tossed simultaneously (at once).

You are required to calculate the probability of getting 5 of the dice and the head on the coin.

- The probability of getting 5 of the dice is $\frac{1}{6}$.
- The probability of getting head on the coin is $\frac{1}{2}$.
- $\therefore$ The probability of getting 5 of the dice and head on the coin $= \frac{1}{6} \times \frac{1}{2}$
 $= \frac{1}{12}$

4.2 Statistical Measures

4.2.1 Mean:

The mean is the average value of a collection of numbers. It is a measure of central tendency of probability distribution. The arithmetic mean is denoted by " $\bar{x}$ " (x bar)

Eg: The demand for a product prevailed for a period of 20 days is as follows.

3	12	7	17	3
14	9	6	11	10
1	4	19	7	15
6	9	12	12	8

If the arithmetic mean of daily demand = $\bar{x}$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{185}{20} = \underline{\underline{9.25 \text{ Units}}}$$

The arithmetic mean is merely an average representation of the demand on each of the 20 days.

Arithmetic mean of data in a frequency distribution

For the previous example, the frequency distribution could be shown as follows;

<u>Daily demand (x)</u>	<u>Frequency (f)</u>	<u>fx</u>
1	1	1
3	2	6
4	1	4
6	2	12
7	2	14
8	1	8
9	2	18
10	1	10
11	1	11
12	3	36
14	1	14
15	1	15
17	1	17
19	1	19
	<u>$\sum f = 20$</u>	<u>$\sum fx = 185$</u>

$$\therefore \bar{x} = \frac{\sum fx}{\sum f} = \frac{185}{20} = \underline{\underline{9.25}}$$

4.2.2 Mode

This is also referred to as "modal value". This is the most frequently occurring value. i.e. The mode is the value that appears most often in a set of data values. (Most common number / value in a data set)

In the previous example; the mode is 12 units, because it is the value / number which occurs most frequently. (03 times/03 days)

4.2.3 Median

This is the middle item of a distribution. The median of a set of ungrouped data can be found by arranging them in ascending or descending order of value, and selecting the item exactly in the middle of the range. (once the set of data is arranged in ascending or descending order of value, such a set is called an array). Therefore, the median is the value of the middle number of an array. The middle item of an odd number of items is calculated as follows.

$$\text{Middle no} = \left(\frac{n+1}{2} \right)_{\text{th}}$$

To calculate the middle item of an even number of items, we have to take the arithmetic mean of the two middle numbers.

In the above example, since 20 items are there, mean/average of the 10th and 11th numbers should be taken as the median.

10th number is 9

11th number is also 9

$$\therefore \text{Median is; } \frac{9+9}{2} = 9$$

4.2.4 Variance and Standard Deviation

The variance (σ^2) is the average of the squared differences from the mean.

Calculation of the variance

- | | | |
|-------|--|--|
| (i) | Difference between value "x" and " $\bar{x}$ " | $\Rightarrow x - \bar{x}$ |
| (ii) | Square of the difference | $\Rightarrow (x - \bar{x})^2$ |
| (iii) | Sum of the squares of the difference | $\Rightarrow \sum (x - \bar{x})^2$ |
| (iv) | Average of the sum | $\Rightarrow \frac{\sum (x - \bar{x})^2}{n}$ |

$$\text{Variance } (\sigma^2) = \frac{\sum (x - \bar{x})^2}{n}$$

Standard Deviation:

This is the square root of the variance. The standard deviation measures the spread of data around the mean. In general, the larger the standard deviation value, in relation to the mean, the more dispersed the data.

The standard deviation, which is the square root of the variance, is a very important measure of spread used in statistics.

Standard deviation for ungrouped data

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

OR

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

Standard deviation for grouped data

$$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

OR

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left[\frac{\sum fx}{\sum f} \right]^2}$$

Example:

The hours of overtime worked in a particular month by 60 employees of ABC PLC are as follows.

<u>Class (Hours)</u>	<u>Frequency (f)</u>
0 < 10	3
10 < 20	6
20 < 30	11
30 < 40	15
40 < 50	12
50 < 60	7
60 < 70	6
	<u>60</u>

To calculate the variance and the standard deviation of the frequency distribution.

Answer

<u>Midpoint (x)</u>	<u>f</u>	<u>fx</u>	<u>x²</u>	<u>fx²</u>
5	3	15	25	75
15	6	90	225	1,350
25	11	275	625	6,875
35	15	525	1225	18,375
45	12	540	2025	24,300
55	7	385	3025	21,175
65	6	390	4225	25,350
	$\sum f = 60$	$\sum fx = 2,220$		$\sum fx^2 = 97,500$

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{2220}{60} = \underline{37}$$

$$\therefore \text{Variance } (\sigma^2) = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$$

$$= \underline{256}$$

$$\therefore \text{Standard deviation } (\sigma) = \sqrt{256}$$

$$= \underline{16 \text{ hours}}$$

4.2.5 Coefficient of variation

The spread of two distributions can be compared using the coefficient of variation.

$$\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Mean}}$$

$$\text{COV} = \frac{\sigma}{\bar{x}} \quad \text{or}$$

$$\text{CV} = \frac{\sigma}{\mu}$$

The bigger the coefficient of variation, the wider the spread/dispersion.

Example:

Two sets of data, A and B, have the following means and standard deviations.

	A	B
Mean	240	260
Standard deviation	50	52
∴ Coefficient of variation	0.208	0.2

Although B has a higher standard deviation, its relative spread is less than A's spread, since the coefficient of variation is smaller.

In probability theory and statistics, the coefficient of variation, also known as the "relative standard deviation" is a standardized measure of dispersion of a probability distribution or a frequency distribution. It is often expressed as a percentage. It can further be defined as the ratio of the standard deviation to the mean.

(5) Decision making under Risk and Uncertainty

As we have already explained in this chapter, "Risk" and "Uncertainty" are two different concepts.

Comparison

	Risk	Uncertainty
(1)	Risk is the probability of winning or losing something worthy.	Uncertainty implies a situation where the future events are not known.
(3)	It can be measured (calculable)	It can not be measured (not calculable)
(3)	Chances of outcomes are known	Outcomes are not known
(4)	Controllable	Uncontrollable
(5)	Minimization – Possible	Minimization – Impossible
(6)	Probabilities can be assigned	Probabilities can not be assigned
(7)	Can be insured	Can not be insured

In general risky projects/risky investments means the projects which have future cashflows that are likely to be variable. The greater the variability the higher the risk. The cash flows are variable, because the estimates of capital expenditure and returns might be for several years ahead, as such, costs and returns may vary from the estimates.

i.e. Costs may escalate (increase rapidly) and the returns may plunge (falling down quickly / dropping quickly)

5.1 Techniques used for decision making under risk and uncertainty

5.1.1 Expected Values (EV)

The expected value (EV) is a weighted average value, based on probabilities. Although the outcome of a decision may not be certain, there is some likelihood of occurring them. Therefore the probabilities could be assigned to the various possible outcomes from an analysis of previous experience.

Expected Value (EV) can be computed as follows.

$$\text{Expected Value (EV)} = \sum np$$

Where; $\sum$ = Sum
 n = Outcomes
 p = probability of occurring outcomes

Example (1)

ABC PLC manufactures and sells product P. The daily sales of product P and the probabilities of occurring them are as follows.

Sales Quantity (n)	Probability (P)
12,000	0.1
20,000	0.2
30,000	0.3
40,000	0.2
45,000	0.2
	1.0

You are required to calculate **the expected daily sales**.



Example (2)

PQR PLC is evaluating two mutually exclusive options A and B, and the probable outcomes of each option are as follows.

Option A		Option B	
Profit	Probability	Profit	Probability
(20,000)	0.1	30,000	0.2
30,000	0.2	90,000	0.4
60,000	0.3	100,000	0.3
80,000	0.2	120,000	0.1
100,000	0.2		
	1.0		1.0

You are required to evaluate the options based on EV.

Example (3)

TOYTO Kandy (Pvt) Ltd., manufactures and sells a product, product R. The estimates of unit selling price, sales demand and variable cost together with their probabilities are as follows.

S/Price		Monthly Demand		Variable Cost	
12	0.1	150,000	0.3	6	0.2
15	0.6	200,000	0.5	7	0.3
18	0.3	250,000	0.2	8	0.4
				9	0.1
	1.0		1.0		1.0

Per unit variable costs do not vary with the volume of sales. Monthly fixed costs would be Rs. 600,000.

You are required to compute the expected profit and advise the management accordingly.

Expected value and decision making

The concept of probability and expected value (EV) are vital in business decision making. Accordingly, the EV can be used for decision making in two ways.

- (1) When there is only one option (as in the above example 3), the project should be accepted if the expected value (EV) is positive and it should be rejected if the EV is negative.
- (2) When there is a choice (two or more options), the option which has the highest EV of profit (or the lowest EV of costs) should be selected.

Example (4)

PDF PLC is evaluating two mutually exclusive projects, for which following possible profits have been estimated.

Project P		Project Q	
Profit (Rs.)	Probability	Profit (Rs.)	Probability
400,000	0.8	(200,000)	0.1
600,000	0.2	500,000	0.2
		700,000	0.6
		800,000	0.1
	1.0		1.0

You are required to assess the projects based on EV.

Expected value of perfect information

Perfect information is said to be available, when 100% accurate prediction can be made about the future. Imperfect information, on the other hand, is not 100% accurate but provides more knowledge than no information.

In decision theory, the expected value of perfect information (EVPI) is the price, that one would be willing to pay in order to gain access to perfect information.

The expected value of perfect information (EVPI) is the difference between the expected value of profit with perfect information and the expected value of profit without perfect information.

Accordingly;

$$EVPI = EVPP - EVUU$$

Where;

EVPI	= Expected value of perfect information
EVPP	= Expected value using a perfect prediction
EVUU	= Expected value under uncertainty

Example:

Ceylon Manufactures Limited (CML) is evaluating three mutually exclusive projects and it has to decide which of the three projects should be undertaken. The top management of CML believes that the success of the projects will depend on consumers' reaction.

There is a 25% chance that consumers' reaction will be strong, a 40% chance that consumers' reaction will be good and a 35% chance that consumers' reaction will be weak.

The CML hopes to use the expected value (EV) concept to evaluate these options. The net present values for each of the possible outcomes are as follows.

	Rs. Million		
Reaction	Project P	Project Q	Project R
Strong	12	16	14
Good	6	4	9
Weak	3	2	1

A market research company believes that, it can provide perfect information on consumer reactions.

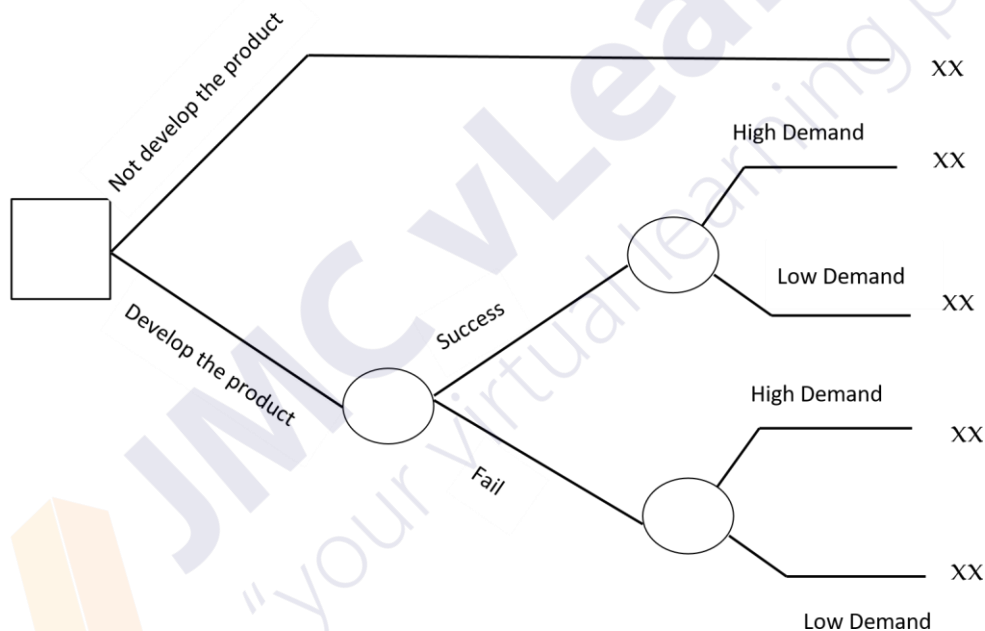
You are required to calculate the maximum amount that should be paid for the information provided by the market research company.

Limitations (Disadvantages) of EV

Evaluating decisions by using EV has a number of limitations.

- (1) The probabilities used to calculate EV are always estimates. They may, therefore, be unreliable or inaccurate. (assigning probabilities to events is highly subjective)
- (2) Expected values are always long term averages and may not be suitable for use in situations involving one-off decisions. They may, therefore, be useful just as a guide to decision making.
- (3) EV does not consider the risk of the attitudes of the people involved in the decision making process. ie. it does not take in to account all of the factor affecting the decisions.
- (4) The time value of money may not be taken in to account.

5.1.2 Decision Trees



As shown in the above diagram, a decision tree is an illustrative (pictorial) method of showing a series of inter related decisions and their expected results. This can incorporate both the probabilities and values of expected results.

For example, we may have to decide whether a new product is developed or not? The decision may be dependent on more than one uncertain variables.

There are two stages involved in decision trees.

(i) **Construction of the decision trees:**

Here, the decision tree is drawn and all of the probabilities and financial outcomes/values are put on the tree.

(ii) **Evaluation and recommendation**

Here, the decision tree is “rolled back” by calculating all the expected values for each outcome points and make the decision by working back across the decision tree. The course of action is then recommended for the Management.

A decision tree should always be drawn, starting from the left hand side of the page and moving across to the right.

Decision Points:

Represents the alternative courses of action that are available. They are within the entity's control (ie. controllable). An entity can either take one course of action or take another. In a decision tree, squares () are used to represent decisions/decision points.

Out comes / Events:

Are not within the control of the entity. They are dependent on the external environment. (For example, customers, suppliersetc). In a decision tree, circles (○) are used to represent outcomes.

- Both decision points and outcome points on a decision tree are always followed by branches.
- When a decision tree is evaluated, the evaluation should start from the right hand side of the tree and should move across to the left. (i.e. in the opposite direction of the tree).

Example:

JMC PLC is evaluating a decision on whether to develop and launch a new product. R & D Department of the company has confirmed that the development cost would be Rs. 400,000. There is a 70% chance that the product launch will be successful and a 30% chance that it will fail.

If it is successful, the level of expected profit and the probabilities of occurring them have been estimated as follows. If the product launch is successful, the product's popularity would be high, medium or low.

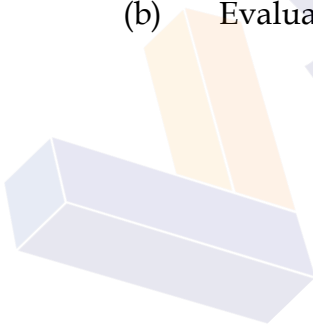
Product's Popularity	Probability	Profits (Rs.)
High	0.2	500,000
Medium	0.5	400,000
Low	0.3	300,000

The above profits are for one year period and the new product will last for 2 years.

If the product launch fails, there is a 0.6 probability that the R & D work (proposal) can be sold for Rs. 50,000 and a 0.4 probability that it will be thrown away with no value.

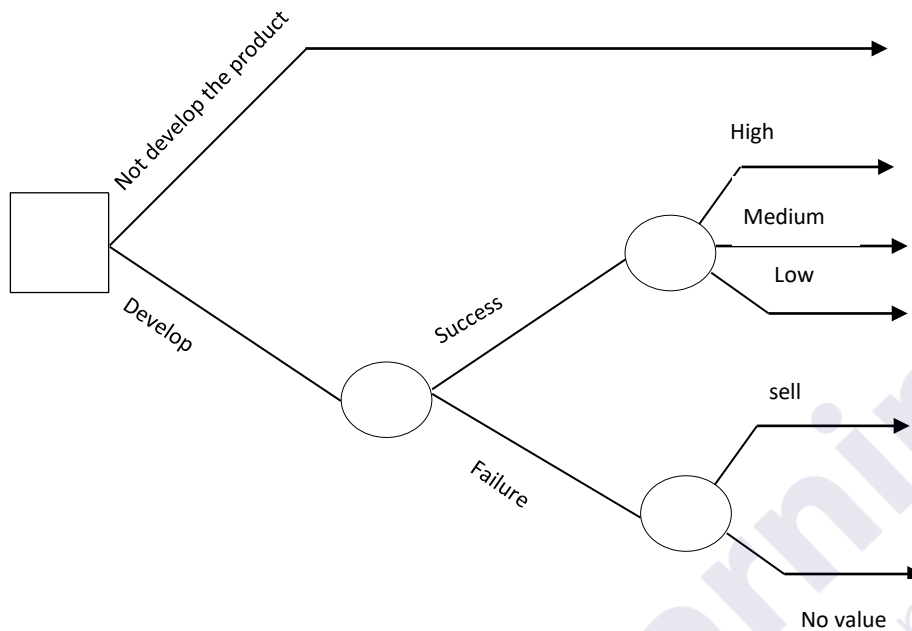
You are required to:

- Draw a decision tree
- Evaluate the decision

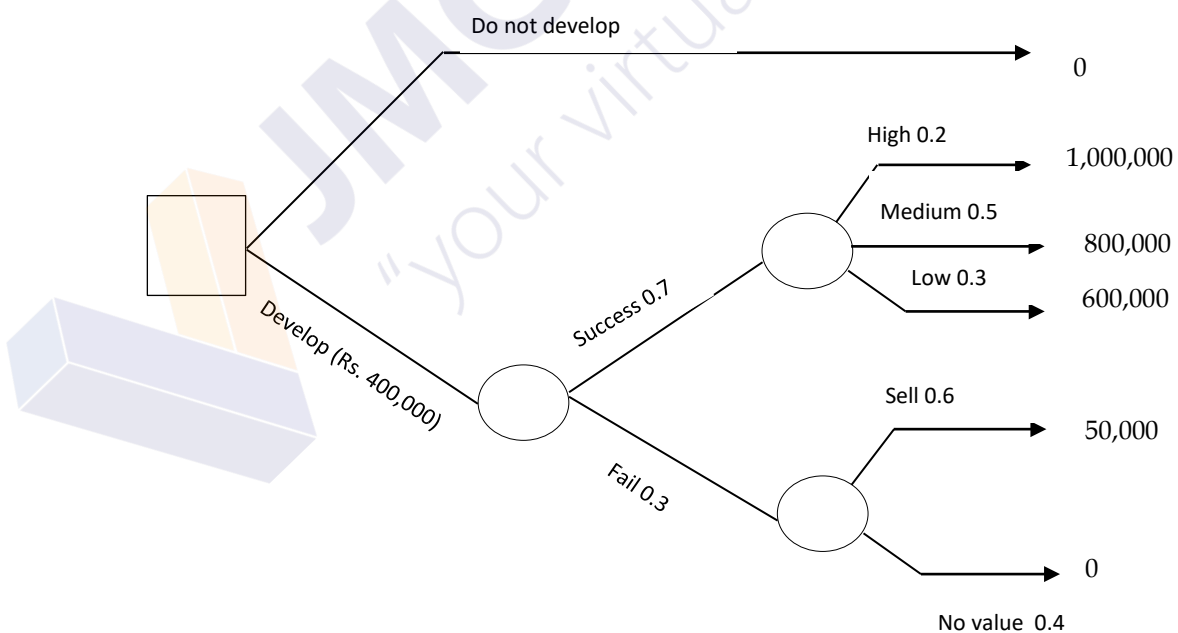


Answer

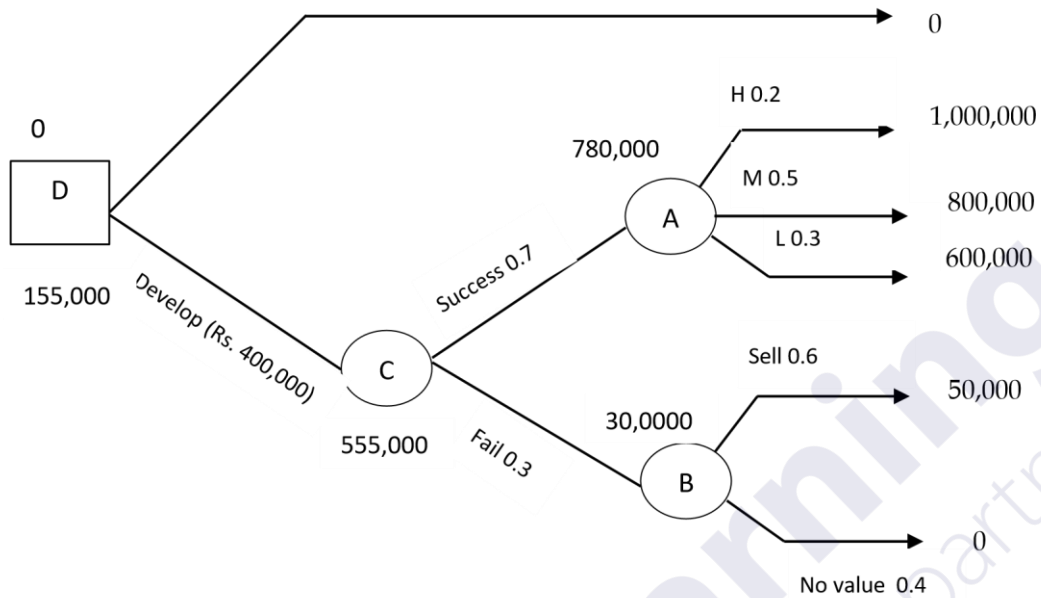
Step (1): The basic structure of the decision tree must be drawn as shown below.



Step (2): Next, the probabilities and the profit figures should be put on. The profits from a successful launch last for two years, then, they must be doubled.



Step (3): Evaluate the points. The decision points and the outcome points must be labeled, starting from the right side and moving across to the left.



Step (4): Calculate expected values at each of the outcome points and the decision points as follows.

$$\begin{aligned} \text{EV at point (A)} &= (1,000 \times 0.2) + (800 \times 0.5) + (600 \times 0.3) \\ &= \underline{780,000} \end{aligned}$$

$$\begin{aligned} \text{EV at point (B)} &= (50 \times 0.6) + (0 \times 0.4) \\ &= \underline{30,000} \end{aligned}$$

$$\begin{aligned} \text{EV at point (C)} &= (780 \times 0.7) + (30 \times 0.3) \\ &= \underline{555,000} \end{aligned}$$

$$\text{EV profit, if not developed} = \underline{0}$$

$$\begin{aligned} \text{EV profit, if developed} &= 555,000 - 400,000 \\ &= \underline{155,000} \end{aligned}$$

Recommendation / Conclusion

It is advisable the management to develop the product since it has an expected profit of Rs. 155,000.

5.1.3 Pay-off tables

Diagrams drawn under decision trees can become unmanageable for complex decisions. A pay-off table can be used as a useful method/way to represent and analyse a scenario with a range of decisions and possible outcomes. These decisions and possible outcomes can be presented in a table format called “Payoff tables”.

- (1) A pay-off table consists of ,
- * Rows - Decisions
 - * Columns - Possible outcomes/events

- (2) Corresponding entries of a pay-off table are called “Pay-off values”.

Events → ↓ Decisions	(a)	(b)
(1)	(1)(a)	(1)(b)
(2)	(2)(a)	(2)(b)

How to make decisions using pay-off tables

The final decision could be based on one of the following criterias.

- (1) Maximax rule
- (2) Maximin rule
- (3) Minimax regret rule

Example:

Soya cream house (SCH) is an ice cream seller. The company is to decide the size of the order to be placed, a small, medium or a large order. In making this decision,

the company should take the weather condition in to consideration. The weather condition would be forecasted as cold, warm or hot. There are nine (09) possible combinations of order size and the weather condition. The pay-off values (profits) for each of the combination are shown below.

Rs. 000

W/Condition Order size	Cold	Warm	Hot
Small	25	20	15
Medium	20	50	30
Large	10	30	75

You are required to determine maximax, maximin and minimax regret solutions.

Answer (With theory)

The highest profit/ pay-off value for each order occurs when the order size is most appropriate (matched/compatible) for the weather. ie; small order in cold weather, medium order in warm weather, large order in hot weather. In other scenarios, profits are low due to either unsold ice cream or lost potential sales. We should determine the decision that the ice cream seller has to make, using each of the decision criteria.

(1) Maximax Criteria

This is also referred to as “Optimism Criteria” or “aggressive criteria”.

The maximax pay off criteria is an optimistic criteria which can be used for decision making as follows.

- Find the maximum value for each decision.

Condition Order size	Cold	Warm	Hot
Small	25	20	15
Medium	20	50	30
Large	10	30	75

- Choose the decision/action that has the highest of the maximum values.
(Maximum of the maximum)

Event O/S	C	W	H
S	25	20	15
M	20	50	30
L	10	30	75

Conclusion

The company will decide upon a large order. The highest pay off value/profit is Rs. 75 Mn.

This criteria is based upon a risk seeking (optimistic) approach. This criteria is suitable for a risk seeking decision maker who presumes that good things always happen.

(2) Maximin Criteria

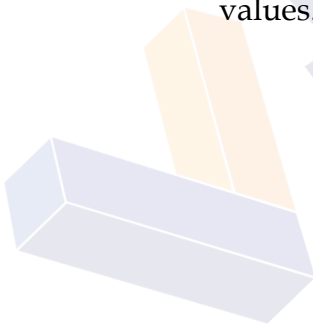
This is also referred to as “Pessimism Criteria” or “Conservative criteria”.

This criteria is a pessimistic criteria which can be used for decision making as follows.

- Find the minimum value for each decision.

Evening O/S	C	W	H
S	25	20	15
M	20	50	30
L	10	30	75

- Choose the decision / action that has the highest value out of the minimum values. (Maximum of minimum)



Events O/S	C	W	H
S	25	20	15
M	20	50	30
L	10	30	75

Conclusion

The company will decide upon a medium order. The pay off value is Rs. 20 Mn.

This criteria is suitable for a risk averse decision maker who presumes that bad things will always happen. (worst case scenario)

(3) Minimax regret criteria

This is also referred to as “opportunity loss criteria”.

This approach/criteria attempt to minimize the regret from making the wrong decision and is based upon first identifying the optimal for each of the outcomes/events.

The decision is made on the basis of the lowest regret.

- Take the maximum value in each events column.

Ev W/C O/S	C	W	H
S	25	20	15
M	20	50	30
L	10	30	75

- Subtract all the values in that column from it.

Ev W/C O/S	C	W	H
S	0	30	60
M	5	0	45
L	15	20	0

- Choose the maximum value (largest lost) of each decision.

Evening W/C O/S	C	W	H
S	0	30	60
M	5	0	45
L	15	20	0

- Choose the minimum value out of the above maximum values (largest losses)

Evening W/C O/S	C	W	H
S	0	30	60
M	5	0	45
L	15	20	0

Conclusion:

The company will decide upon a large order. The lowest regret value is Rs. 20 Mn.