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Basic Mathematics

AAT Level I

BMS - Business Mathematics and Statistics

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JAYASEKERA MANAGEMENT CENTRE (PVT) LTD
Estd. 1972

JMC Jayasekera Management Centre (Pvt) Ltd
Pioneers in Professional Education
65/2A, Chittampalam Gardiner Mawatha, Colombo 02 | T: +94 112 430451 | E: info@jmc.lk | F: +94 115 377917

Chapter 1

BASIC MATHEMATICS

SET OF REAL NUMBERS

When any number (say K), after squaring the result is either positive or zero ($K^2 \geq 0$) then it is known as a real number inside a real number set there are various subsets.

Examples

* Natural Numbers
* Whole Numbers
* Integer Numbers
* Rational Numbers
* Irrational Numbers

NATURAL NUMBERS

The set of natural numbers denoted by N is the set of counting numbers. The set of natural numbers is also called positive integers.

Example

$N = \{1, 2, 3, 4, \ldots\ldots.\}$

WHOLE NUMBERS

The set of natural numbers together with zero is known as whole numbers. It is denoted by W.

Example

$W = \{0, 1, 2, 3, 4, \ldots\ldots.\}$

SET OF INTEGERS

The natural numbers together with zero and negative integers form the set of integers. It is denoted by Z.

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Example

Positive Integers = $Z^+ = \{1, 2, 3, 4 \ldots\ldots\}$

Negative Integers = $Z^- = \{-1, -2, -3, -4 \ldots\ldots\}$

Integers = $Z = \{\ldots\ldots -3, -2, -1, 0, 1, 2, 3, 4 \ldots\ldots\}$

SET OF RATIONAL NUMBERS

A rational number is one that can be written as a format p/q, where p and q are real numbers and $q \neq 0$. It is denoted by Q.

Example

* Set of all positive or negative integers

  $5 = \frac{5}{1}$ $\quad -3 = \frac{-3}{1}$

* Set of all terminate decimals

  $0.5 = \frac{1}{2}$ $\quad -0.25 = \frac{-1}{4}$

* Set of all repeating decimal

  $0.3333 = \frac{1}{3}$ $\quad -0.666 = \frac{-2}{3}$

SET OF IRRATIONAL NUMBERS

Numbers represented by non-terminating non-repeating are called irrational numbers.

Example

$\sqrt{2} = 1.4142\ldots\ldots$

$\sqrt{3} = 1.73205\ldots\ldots$

$\pi = 3.14159265$

INDICIES (EXPONENT)

$$\overset{\text{Index}\downarrow}{\mathbf{2}^{3}} = \mathbf{8} \leftarrow \text{Number}$$

$$\underset{\text{Base}}{\uparrow}$$

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Definition of an Exponent

If r is any real number and n is a positive integer an exponent r^n can be defined as follows.

* $r^n = \underbrace{r\times r\times r\times r\times \ldots\times r}_{\text{n time}}$
* $r^{-n} = \dfrac{1}{r^n} = \dfrac{1}{\underbrace{r\times r\times r\times \ldots\ldots\times r}_{\text{n - time}}}$
* $r^n = 1$

Laws of Indies

* $x^m \times x^n = x^{m+n}$
* $x^m / x^n = x^{m-n}$
* $(x^m)^n = x^{mn}$

Standard Form of a Number

Any number can be expressed as a value between 1 and 10 multiplied by 10. A number expressed in this way is known to be in standard form.

Examples

* $764.9 = 7.649 \times 10^2$
* $0.0003467 = 3.647 \times 10^{-4}$
* $0.897 = 7.97 \times 10^{-1}$
* $1456.9 = 1.4569 \times 10^3$

Percentages

One way of comparing fractions is to express them with a denominator of 100 fractions which are expressed with a denominator of 100 are said to be percentages.

Examples

* $\dfrac{3}{5} = \dfrac{60}{100} = 60\%$
* $\dfrac{1}{4} = \dfrac{25}{100} = 25\%$
* $\dfrac{3}{20} = \dfrac{15}{100} = 15\%$

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Percentage Profit and Loss

When a trader buys or sells goods, the cost price is the price at which each item is bought and the selling price is the price all which each item is sold profit or loss percentage may be calculate on cost price or selling price.

The relationship between the cost and selling price can be established as follows;

Cost + Profit = Selling Price

As a Mark-up | As a Margin

Percentage profit calculate on cost price is known as mark-up.

$$\text{Mark up} = \frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100$$

Percentage Profit calculate on selling price is known as margin.

$$\text{Mrgin} = \frac{\text{Selling Price} - \text{Cost Price}}{\text{Selling Price}} \times 100$$

Sales Tax or Value Added Tax

Sales tax or value added tax (VAT) is a tax on goods and service which are bought to be paid to the buyer. The rate of tax may vary from time to and also from goods to goods.

Worked Examples

Example ①

A customer paid \$ 230 for a television which had been reduced by 15%. Calculate the original price of television before the price reduction.

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Answer

Let the original price as x
Given that,

$0.85x = 230$

$$x = \frac{230}{0.85}$$

$\underline{x = 271}$

Example ②

In calculating charge to the wholesaler, a furniture manufacturer adds 35% profit loading to his cost of production.

The wholesaler adds 20% to determine his charges to retailer who in turn add 25% to calculate his normal selling price.

A customer who bought an article of furniture during an annual sale period was allowed Rs. 150 deduction and paid cash Rs. 12,000.

Calculate manufacture's cost on production.

Answer

Customer

Cash paid	Rs. 12,000
Discount	Rs. 150
Selling price	Rs. 12,150

Retailer

Selling price (125%)	= 12,150
Purchasing price (100%)	= 12,150 ÷ 1.25
	= Rs. 9,720

Wholesaler

Purchasing price (120%)	= Rs. 9,720
Manufacture's selling price	= 9,720 ÷ 1.2
	= Rs. 8,100

Manufacturer

Manufacture's selling price (135%)	= 8,100
Manufacture's cost	= 8,100 ÷ 1.35
	= Rs. 6,000

Exercise 01

A refrigerator was sold for Rs. 45,500 yielding 30% profit on cost. Calculate selling price in order to yield only a 10% profit on cost.

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Exercise 02

A year ago retail price of Y was Rs. 50 exclusive of vat at 15%. The current price of Y to the consumer inclusive of 30% VAT, is now doubled that of last year. Calculate current price of Y exclusive of VAT.

Algebraic Expressions

A collection of variable values and constant values can be defined as an algebraic expression.

Example

* $4x + 5$
* $3x^2 - 12x + 7$
* $6x + 5y - 9$
* $x^2 - 2y^2 + 11xy$

Equations

If any two algebraic expressions are connected by using an equal sign, the result obtained is known as an equation.

Example

* $2x + 3 = 5-x$ (Simple Linear Equations)
* $x^2 + 2x + 1 = 0$ (Quadratic Equations)
* $\left.\begin{array}{l} 3x + 4y = 12 \\ x - y = 5 \end{array}\right\}$ Simultaneous Linear Equations with two variables

Solving Simple Linear Equations

Worked Examples

Example ①

$$\begin{aligned} 7x - 5 &= 5x - 8 \\ 7x - 5x &= -8 + 5 \\ 2x &= -3 \\ x &= -3/2 \end{aligned}$$

Example ②

$$\begin{aligned} 4(3x - 1) &= 10x + 8 \\ 12x - 4 &= 10x + 8 \\ 12x - 10x &= 8 + 4 \\ 2X &= 12 \\ X &= 6 \end{aligned}$$

Example ③

$$
\begin{aligned}
7x - 2\{5x - 3(x - 2)\} &= 15 \\
7x - 2\{5x - 3x + 6\} &= 15 \\
7x - 2\{2x + 6\} &= 15 \\
7x - 4x - 12 &= 15 \\
3x - 12 &= 15 \\
3x &= 15 + 12 \\
3x &= 27 \\
x &= 9
\end{aligned}
$$

Example ④

$$\frac{3x}{2} - \frac{2x-1}{3} = 1 - \frac{x-1}{4}$$

Common D. 12

$$\frac{18x}{\cancel{12}} - \frac{4(2x-1)}{\cancel{12}} = \frac{12}{\cancel{12}} - \frac{3(x-1)}{\cancel{12}}$$

$$
\begin{aligned}
18x - 4(2x - 1) &= 12 - 3(x - 1) \\
18x - 8x + 4 &= 12 - 3x + 3 \\
10x + 4 &= 15 - 3x \\
10x + 3x &= 15 - 4 \\
13x &= 11 \\
x &= 11/13
\end{aligned}
$$

Exercises

① $5(2x - 3) = 6x + 5$

② $4\{3x - 2(x - 1)\} = x + 5$

③ $\frac{3x-4}{5} = \frac{x+1}{3}$

④ $\frac{4x-3}{x+1} = \frac{4}{3}$

⑤ $\frac{2x}{3} - \frac{x-2}{4} = \frac{x+1}{2}$

⑥ $4[3x - 2\{4x - 3(x-2)\}] = 12$

⑦ $\frac{4x}{5} - \frac{3x-2}{4} = 3 + \frac{x+1}{2}$

⑧ $\frac{x+5}{3} - 2 = \frac{3x}{5} + 1$

Quadratic Equations

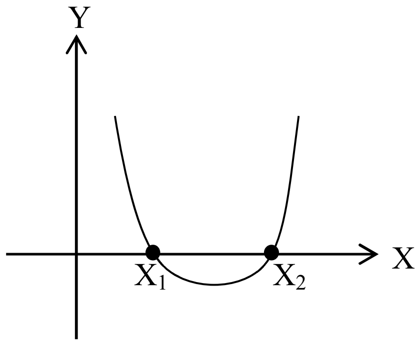
Standard format f a quadratic equation is $ax^2 + bx + c = 0$ where a, b and c are real numbers and $a \neq 0$.

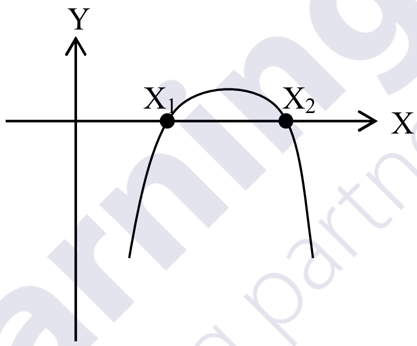
Examples

* $3x^2 - 5x + 8 = 0$ (a = 3 B = -5 c = 8)
* $-x^2 + 8x = 0$ (a = -1 B = 8 c = 0)
* $x^2 + 17 = 0$ (a = 1 B = 0 c = 17)
* $5x^2 = 0$ (a = 5 B = 0 c = 0)

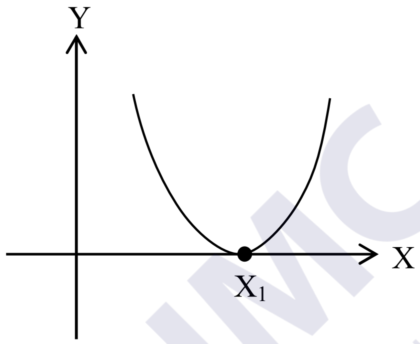
There are 3 types of solutions available in quadratic equations as follows.

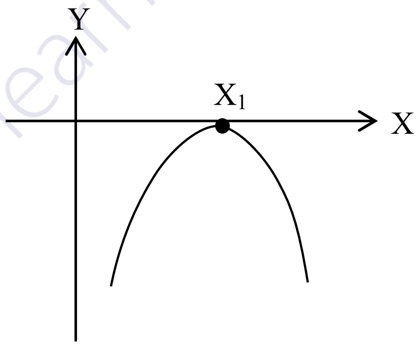
* Two different real value solutions.



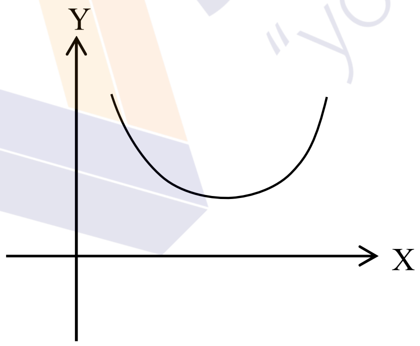


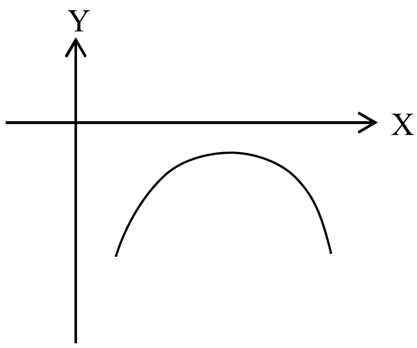
* Only one real value solutions.





* No real value solutions.





Solving Quadratic Equations By Using Formula

Standard format of a quadratic equation is $ax^2 + bx + c = 0$ solutions of the above quadratic equation is given by a following formula.

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Worked Examples

Example ①

$x^2 - 6x + 8 = 0$

$*a = 1 \quad *b = -6 \quad *c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 32}}{2}$$

$$x = \frac{6 \pm \sqrt{4}}{2}$$

$$x = \frac{6 \pm 2}{2}$$

$$x = \frac{6 + 2}{2} \text{ or } x = \frac{6 - 2}{2}$$

$\underline{x = 4 \text{ or } x = 2}$

(Two different real value solutions)

Example ②

$x^2 + 3x = 1$

$x^2 + 3x - 1 = 0$ (Convert into a standard format)

$*a = 1 \quad *b = 3 \quad *c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{13}}{2}$$

$$x = \frac{-3 \pm 3.61}{2}$$

$$x = \frac{3 + 3.61}{2} \text{ or } x = \frac{3 - 3.61}{2}$$

$\underline{x = 3.305 \text{ or } x = -0.305}$

(Two different real value solutions)

Example ③

$x^2 = 2x - 1$
$x^2 - 2x + 1 = 0$ (Convert into a standard format)
$*a = 1 \quad *b = -2 \quad *c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{4 - 4}}{2}$$
$$x = \frac{2 \pm \sqrt{0}}{2}$$
$$x = \frac{2 \pm 0}{2}$$
$$x = \frac{2}{2}$$
$\underline{\underline{x = 1}}$

(Only one real value solutions)

Example ④

$5x^2 + x + 1 = 0$
$*a = 5 \quad *b = 1 \quad *c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(5)(1)}}{2(5)}$$
$$x = \frac{-1 \pm \sqrt{1 - 20}}{10}$$
$$x = \frac{-1 \pm \sqrt{-19}}{10}$$

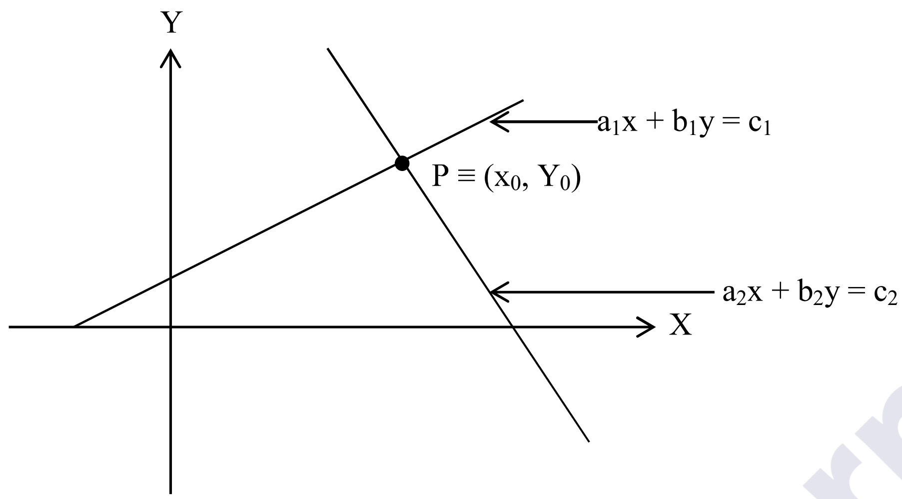
(No real value solutions)

Exercises

① $x^2 - 7x + 10 = 0$
② $2x^2 = 5x + 8$
③ $x^2 = 6x - 9$
④ $x^2 + x + 1 = 0$
⑤ $3x^2 = 6x$
⑥ $x^2 + 25 = 0$

Simultaneous Liner Equations with 2 variables

Simultaneous Liner Equations with 2 variables are representing straight lines on XY coordinate axes. Solutions of the above set of equations are an intersection point of two lines.



Solution $P \equiv (x_0, Y_0)$

Worked Examples

Example ①

$2x + 3y = 13 \longrightarrow$ ①

$x - 3y = 2 \longrightarrow$ ②

① + ②

$$
\begin{aligned}
(2x + 3y) + (x - 3y) &= 13 + 2 \\
2x + 3y + x - 3y &= 15 \\
2x + x &= 15 \\
3x &= 15 \\
x &= 5
\end{aligned}
$$

Sub x = 5, to ①

$$
\begin{aligned}
x - 3y &= 2 \\
5 - 3y &= 2 \\
-3y &= 2 - 5 \\
-3y &= -3 \\
y &= \frac{-3}{-3} \\
y &= 1
\end{aligned}
$$

Solution (5, 1)

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<u>Example ②</u>

$3x - 2y = 8 \longrightarrow$ ①
$x - y = 3 \longrightarrow$ ②
① × 2 $\quad 2x - 2y = 6 \longrightarrow$ ③

① - ③
$(3x - 2y) - (2x - 2y) = 8 - 6$
$3x - 2y - 2x + 2y = 2$
$3x - 2x = 2$
$x = 2$

Sub $x = 2$, to ②
$x - y = 3$
$2 - y = 3$
$-y = 3 - 5$
$-y = 1$
$y = 1$

<u>Solution (2, -1)</u>

<u>Example ③</u>

$4x - 3y = 26 \longrightarrow$ ①
$3x - 2y = 19 \longrightarrow$ ②
① × 2 $\quad 8x - 6y = 52 \longrightarrow$ ③
② × 3 $\quad 9x - 6y = 57 \longrightarrow$ ④

④ - ③
$(9x - 6y) - (8x - 6y) = 57 - 52$
$9x - 6y - 8x + 6y = 5$
$9x - 8x = 5$
$x = 5$

Sub $x = 5$, to ①
$4x - 3y = 26$
$4(5) - 3y = 26$
$20 - 3y = 26$
$-3y = 26 - 20$
$-3y = 6$
$y = -2$

<u>Solution (5, -2)</u>

<u>Exercises</u>

❶ $2x + y = 23$
$4x - y = 19$

❷ $x + 2y = 4$
$2x + 3y = 7$

❸ $11x + 8y = 31$
$13x - 6y = 83$

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Word Applications of Simultaneous Linear Equations

Worked Examples

Example ①

Zobo Ltd. Manufactures smoking pipes using two machine processes P_1 and P_2. In a working week there are 200 hours of P_1 process time available, but 130 available hours for P_2. Two particular models of 'A' and 'B' needs 15 minutes of P_1 and 12 minutes of P_2.

One unit of 'A' uses 20 minutes of P_1 process and 10 minutes of P_2 process. One unit of 'B' needs 15 minutes of P_1 and 12 minutes of P_2.

Calculate combination of quantities of 'A' and 'B' which use up all the available P_1 process and all available of P_2 process. You may assume a mix of both 'A' and 'B' will be produced.
Solution

Let the number of units produced by model 'A' = x
model 'B' = y

* For Process P1

$200 \times 60 = 20x + 15y$
$12000 = 20x + 15y$
$20x + 15y = 12000$

* For Process P2

$130 \times 60 = 10x + 12y$
$7800 = 10x + 12y$
$10x + 12y = 7800$

$20x + 15y = 12000 \longrightarrow$ ①
$10x + 12y = 7800 \longrightarrow$ ②

We can solve the above pairs of equations by using calculator

$a_1 = 20$	$a_2 = 10$
$b_1 = 15$	$b_2 = 12$
$c_1 = 12000$	$c_2 = 7800$

$x = 300$
$y = 400$

No: of units produced by model 'A' = 300
No: of units produced by model 'B' = 400

Exercise 01

A company makes two models of CD players known as 'Basic' and 'Advanced' models. Daring the year 2012 it made 4000 units of Basic model and 3000 units of Advanced model, and its production costs totaled 27 million rupees.

Daring 2013, it made 3000 units of Basic model and 5000 units of Advanced model and its production cost were 34 million rupees.

Calculate the production cost of a Basic model and for an Advanced model by using simultaneous equations.

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Chapter 2

PROGRESSIONS

* Sequence
* Arithmetic Progressions
* Geometric Progressions

SEQUENCE

A Sequence is a set of values, in which there is general trend or pattern of Growth or Decline.

Example:-

If the monthly production of a factory in the last six months (in 000units) was as follows.
200, 240, 280, 320, 360, 400

A sequence may or may not terminate at some point if it does, it is known as a finite sequence otherwise it is known as an infinite sequence.

General Term of a Sequence

The n^{th} term of a sequence (denoted by T_n) is called the general term of a sequence. If the general term of a sequence is given the term of a sequence can be determined.

Worked Examples

Write down the first 3 terms of a sequence, whose general term is given below.

a) $T_n = n^2 + 7$

b) $T_n = (-1)^{n+1} \left(\frac{1}{3}\right)^n$

Solution

a) $T_n = n^2 + 7$

$n = 1 \longrightarrow T_1 = (1)^2 + 7 = 1 + 7 = 8$
$n = 2 \longrightarrow T_2 = (2)^2 + 7 = 4 + 7 = 11$
$n = 3 \longrightarrow T_3 = (3)^2 + 7 = 9 + 7 = 16$

<u>8,11,16…..</u>

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b) $$T_n = (-1)^{n+1} \left(\tfrac{1}{3}\right)^n$$

$$n = 1 \longrightarrow T_1 = (-1)^{1+1} \left(\tfrac{1}{3}\right)^1 = (-1)^2\left(\tfrac{1}{3}\right) = (+1)\left(\tfrac{1}{3}\right) = \tfrac{1}{3}$$

$$n = 2 \longrightarrow 2_1 = (-1)^{2+1} \left(\tfrac{1}{3}\right)^2 = (-1)^3\left(\tfrac{1}{9}\right) = (-1)\left(\tfrac{1}{9}\right) = -\tfrac{1}{9}$$

$$n = 3 \longrightarrow T_3 = (-1)^{3+1} \left(\tfrac{1}{3}\right)^3 = (-1)^4\left(\tfrac{1}{3}\right)^3 = (+1)\left(\tfrac{1}{27}\right) = \tfrac{1}{27}$$

$$\frac{1}{3}, \frac{1}{9}, \frac{1}{27} \ldots.$$

Write down first three terms of the progression whose General terms are given below.

a) $T_n = n\,(n+2)$

b) $T_n = (n^2-3)$

c) $T_n = \dfrac{(n^2+1)}{(2n^2-1)}$

d) $T_n = 4 + 2/n$

In general the n^{th} term of a Progression cannot be determined; However there are two Sequences for which we can find the general term. They are,

- Arithmetic Sequences
- Geometric Sequences

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ARITHMETIC PROGRESSION

When we consider two consecutive terms in a series and subtract the term of the left from the term on right, then if the result is same for any consecutive pair, the series is called an arithmetic progression. The common constant obtained by subtraction known as common difference of the progression. It is denoted by 'd'.

Therefore,

$$\boxed{d = T_2 - T_1 = T_3 - T_2 = T_4 - T_3 \ldots.}$$

Examples

* 5, 8, 11, 14 …… (d = 3)
* 65, 60, 55, 50 ….. (d = -5)
* 4, 9, 14, 19, 24 ….. (d = +5)

Standard Format of an Arithmetic Progression

Consider the following example

4, 9, 14, 19, 24 …..

The above one is an arithmetic progression with first term 4 and common difference 5. Terms of the above progression can be represented in a following way

$T_1 = 4$
$T_2 = 4 + 5 = 9$
$T_3 = 4 + 2(5) = 14$
$T_4 = 4 + 3(5) = 19$

If the first term is denoted by 'a' and common difference is denoted by 'd' then we get the standard form of arithmetic progression as:

$T_1 = a \longrightarrow$ (First Term)
$T_2 = a + d \longrightarrow$ (Second Term)
$T_3 = a + 2(d) \longrightarrow$ (Third Term)
$T_4 = a + 3(d) \longrightarrow$ (Fourth Term)
…………………………....
$T_{12} = a + 11(d) \longrightarrow$ (12^{th} Term)
…………………………
$T_n = a + (n-1)d \longrightarrow$ (n^{th} Term)

Therefore n^{th} term of an arithmetic progression is given by the following formula.

$$\boxed{T_n = a + (n-1)\,d}$$

* $T_n = n^{th}$ Term
* a = First Term
* d = Common Difference
* n = No. of Terms

Sum of First n terms of an Arithmetic Progression

Case I (Last term is given)

$T_1, T_2, T_3, T_4 \ldots\ldots\ldots$ T_n ⟵ Given

$$\boxed{S_n = \frac{n}{2}\{T_1 + T_n\}}$$

* S_n = Sum of First n terms
* n = No. of terms
* T_1 = First term
* $T_n = n^{th}$ term

Case II (Last tem is not given)

$T_1, T_2, T_3, T_4 \ldots\ldots\ldots$ T_n ⟵ Not given

$$\boxed{S_n = \frac{n}{2}\{2a + (n-1)d\}}$$

* S_n = Sum of First n terms
* n = No. of terms
* a = First term
* d = Common Difference

Worked Examples

Example ①

Evaluate (i) 20^{th} term (ii) 75^{th} term (iii) n^{th} term of an arithmetic progression 5, 9, 13, 17…….

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Solution

5, 9, 13, 17…..

(i) $\underline{T_{20} = ?}$

$T_{20} = a + 19d$

$* a = 5 \quad * d = 4$

$T_{20} = 5 + 19(4)$

$T_{20} = 5 + 76$

$\underline{\underline{T_{20} = 81}}$

(ii) $\underline{T_{75} = ?}$

$T_{75} = a + 74d$

$* a = 5 \quad * d = 4$

$T_{75} = 5 + 74(4)$

$T_{75} = 5 + 296$

$\underline{\underline{T_{75} = 301}}$

(iii) $\underline{T_n = ?}$

$T_n = a + (n-1)d$

$* a = 5 \quad * d = 4$

$T_n = 5 + (n-1)(4)$

$T_n = 5 + 4n - 4$

$\underline{\underline{T_n = 4n + 1}}$

<u>Exercise 01</u>

Evaluate (i) 30th term (ii) 50th term (iii) nth term of on arithmetic progression 55, 50, 54, 40…

<u>Example ②</u>

Second and Fifth terms of an arithmetic progression is -19 and -13 respectively.

(i) Evaluate First term and Common Difference
(ii) Write down first four terms of an arithmetic progression
(iii) Calculate 50th term
(iv) Find the sum of first 50 terms of a seris

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Solution

(i) Evaluate 'a' and 'd'

Given that $T_2 = -19$ $\quad T_5 = -13$

But $T_2 = a + d$ and $T_5 = a + 4d$ (From standard Form)

$a + d = -19 \longrightarrow ①$

$a + 4d = -13 \longrightarrow ②$

② - ①

$(a + 4d) - (a + d) = -13-(-19)$

$a + 4d - a - d = 13 + 19$

$4d - d = 6$

$3d = 6$

$\underline{\underline{d = 2}}$

Sub. $d = 2$ to ①

$a + d = -19$

$a + 2 = -19$

$\underline{\underline{a = -21}}$

(ii) First Four Terms

-21, -19, -17, -15…..

(iii) $\underline{T_{50} = ?}$

$T_{50} = a + 49\,d$

$* \; a = -21 \quad * \; d = 2$

$T_{50} = -21 + 49\,(2)$

$T_{50} = -21 + 98$

$\underline{\underline{T_{50} = 77}}$

(iv) $\underline{S_{50} = ?}$

$S_n = \frac{n}{2}\{T_1 + T_n\}$

$* \; n = 50 \quad * \; T_1 = -21 \quad * \; T_n = 77$

$S_{50} = \frac{50}{2}\{-21 + 77\}$

$S_{50} = 25 \times 56$

$\underline{\underline{S_{50} = 1400}}$

<u>Exercise 02</u>

Third and sixth terms of an arithmetic progression is 10 and 22 respectively.

i. Evaluate first term and common difference
ii. Write down first four terms of a series
iii. Calculate 25th term
iv. Find the sum of first 100 terms of a series

Example ③

In an arithmetical progression nth term is equal to $T_n = 7 - 4n$.

(i) Find out first term and common difference

(ii) Which term will be equal to -33

(iii) Find the sum of first 12 terms of a series

Solution

(i) Evaluate 'a' and 'd'

$$T_n = 7 - 4n$$
$$n = 1 \longrightarrow T_1 = 7 - 4(1) = 7 - 4 = 3$$
$$n = 2 \longrightarrow T_2 = 7 - 4(2) = 7 - 8 = 1$$
$$n = 3 \longrightarrow T_3 = 7 - 4(3) = 7 - 12 = -5$$

3, -1, -5…….

The above one is an arithmetical progression with,

$\underline{a = 3 \quad \text{and} \quad d = -4}$

(ii) $\underline{T_{n?} = -33}$

$$T_n = 7 - 4n$$
$$* \; T_n = -33 \qquad * \; n = ?$$
$$-33 = 7 - 4n$$
$$4_n = 7 + 33$$
$$4_n = 40$$
$$\underline{n = 10}$$

(iii) $\underline{S_{12} = ?}$

$$S_n \quad \frac{n}{2}\{2a + (n-1)d\}$$
$$* \; n = 12 \qquad * \; a = 3 \qquad * \; d = -4$$
$$S_{12} = \frac{12}{2}\{2(3) + (12-1)(-4)\}$$
$$S_{12} = 6\{6 + (11)(-4)\}$$
$$S_{12} = 6\{6 - 44\}$$
$$S_{12} = 6(-38)$$
$$\underline{S_{12} = -228}$$

Exercise 03

1. In an arithmetical nth term is equal to $T_n = 4 - n$

   (i) Find out first term and common difference

   (ii) which term will be equal to -20

   (iii) Find the sum of first 50 terms of a series

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2. The sum of first n terms is given by $S_n = 8n^2 - n$

   (i) Show that series is an arithmetic progression and evaluate first term and common difference
   (ii) Calculate n^{th} term of a series
   (iii) Which term will be equal to 263?

Word Applications in an Arithmetic Progressions

Example ④

Find the sum of all integers between 50 and 200 which are divisible by 6

Solution

The series is as follows

54, 60, 66, 72……… 198 ⟵ Last Term

No. of terms of a series

$T_n = a + (n-1)d$

$* T_n = 198 \quad * a = 54 \quad * d = 6$

$198 = 54 + (n-1)(6)$

$198 = 54 + 6n - 6$

$198 = 48 + 6n$

$198 - 48 = 6n$

$150 = 6n$

$25 = n$

Sum of terms

$S_n = \frac{n}{2}\{T_1 + T_n\}$

$* n = 25 \quad * T_1 = 54 \quad * T_n = 198$

$S_{25} = \frac{25}{2}\{54+198\}$

$S_{25} = 25 \times \frac{252}{2}$

$S_{25} = 3150$

Example ⑤

ABC Ltd has obtained an order to supply 530000 units of a product known as “Easy Care” to one of its customers. The customer requested ABC Ltd that final delivery should be within 48 days.

The factory manager of ABC Ltd has estimated that output of Easy Care on day 1 will be 3500 units but the daily output can be increased by 500 units each day by introducing additional resources.

a) Discuss whether it would be possible for ABC Ltd to complete the contract with in specified time period.

b) If the company is unable to increase the output expected, after 20 days, daily production remains constant at the level achieved on day 20, How long would it take to complete the order?

Solution

a) Daily production of Easy Care forms on AP with a = 3500, b = 500, s = 530000

No: of days taken to complete the order = No: of terms of an AP whose sum is equal to 530000

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$530000 = \frac{n}{2}\{2\times 3500+(n-1)(500)\}$$

$$530000 = \frac{n}{2}\{7000+500n-500\}$$

$$530000 = \frac{n}{2}\{6500+500n\}$$

$530000 = 3250n + 250n^2$

$0 = 250n^2 + 3250n - 530000$

$250n^2 + 3250n - 530000 = 0$

a = 250   b = 3250   c = 530000

n = 40   n = -53 (invalid)

It will take 40 days to produce the required output and hence the company could complete the contract.

b) No: of days which is taken to complete the order

Production on 20^{th} day = T_{20}

$T_{20} = a + 19d$

$*$ a = 3500 $*$ d = 500

$T_{20} = 3500 + 19 \times 500$

$T_{20} = 13000$

Total production over first 20 days = S_{20}

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$n = 20 \quad a = 3500 \quad d = 500$

$S_{20} = \frac{20}{2}\{2 \times 3500 + (20-1)(500)\}$

$S_{20} = 10\{7000 + 9500\}$

$S_{20} = 10 \times 16500$

$S_{20} = 165000$

Balance Qty $= 530000 - 165000$
$= 365000$

Time needed to complete order $= 365000/13000 = 28.08 = 29$ Days
$= 20 + 29$
$= \underline{49 \text{ Days}}$

Exercise 04

A person is appointed to a post with an initial salary of Rs. 5600 per month with an annual increment of Rs. 400

(i) Find the monthly salary in the 7th year of his service

(ii) Find the total amount hi can collect from salary when he completes 10 years of his service.

Exercise 05

In a particular second motor car travels 2 meters more than the distance travelled in previous second. If the distance travelled in first second is 3m, calculate the time taken to travel 120m.

Exercise 06

Firm 'A' starts producing 1800 units of an item in the first month and increases production by 60 units every month. Firm 'B' starts producing 3200 units of another item and decrease its production by 40 units. Assuming that production of Firm 'A' grows and production of Firm 'B' decays in an arithmetical progression and assuming that both firms starts producing items during the same month.

Calculate,

(i) The month in which both firms will produce same number of items

(ii) The number of months after which the total production of either firm will have the same value.

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GEOMETRIC PROGRESSIONS

When we consider two consecutive terms in a series and divide the term on right by the term on left, then if the result is same for any consecutive pair, the series is called Geometric Progression. The common constant obtained by the division is known as the common ratio of a progression and it is denoted by ‘r’.

Therefore,

$$r = T_2/T_1 = T_3/T_2 = T_4/T_3 \ldots\ldots$$

Examples:-

* 4, 20, 100, 500…….. (r = 5)
* 3, -6, 12, -24 ………. (r = -2)
* 256, 64, 16, 4 …..…. ($r = \frac{1}{4}$)
* -625, 125, -25, 5…... ($r = \frac{-1}{5}$)

<u>Standard format of a Geometric Progression</u>

Consider the following example.

1, 3, 9, 27, 81……..

The above one is a geometric progression with first term 1 and common ratio 3. Terms of a geometric progression can be represented in a following way.

$T_1 = 1 \longrightarrow$ (First Term)
$T_2 = 1 \times 3 = 3 \longrightarrow$ (Second Term)
$T_3 = 1 \times (3)^2 = 19 \rightarrow$ (Third Term)
$T_4 = 1 \times (3)^3 = 27 \rightarrow$ (Fourth Term)

If the first term is denoted by ‘a’ and common ratio is denoted by ‘r’ then we get the standard form of geometric series as:

$T_1 = a \longrightarrow$ (First Term)
$T_2 = ar \longrightarrow$ (Second Term)
$T_3 = ar^2 \longrightarrow$ (Third Term)
$T_4 = ar^3 \longrightarrow$ (Fourth Term)
…………………….....
$T_{10} = ar^9 \longrightarrow$ (10^{th} Term)
………………………
$T_n = ar^{n-1} \longrightarrow$ (n^{th} Term)

Therefore n^{th} term of a geometric progression is given by the following formula.

$$\boxed{T_n = ar^{n-1}}$$

* $T_n = n^{th}$ Term
* a = First Term
* r = Common Ratio
* n = No. of Terms

Sum of First n terms of an Geometric Progression

$$\boxed{S_n = \frac{a\{r^n - 1\}}{r-1}}$$

* S_n = Sum of First n terms
* n = No. of terms
* a = First term
* r = Common Ratio

Note

If value of r is in between -1 and +1 (-1<1<+1) then sum of an infinite number of terms can be calculated as:

$$\boxed{S_{00} = \frac{a}{1-r}}$$

* S_{00} = Sum of an infinite no: of terms
* a = First term
* r = Common Ratio

Worked Examples

Example ①

a) Evaluate (i) 7^{th} term (ii) 8^{th} term of a geometric progression 2, -6, 18, -54…….

Solution

2, -6, 18, -54…..

(i) $\underline{T_7 = ?}$

$T_7 = ar^6$

$* \; a = 2 \quad * \; r = -3$

$T_7 = 2(-3)^6$

$T_7 = 2 \times 729$

$\underline{T_7 = 1458}$

(ii) $\underline{T_8 = ?}$

$T_8 = ar^7$

$* \ a = 2 \quad * \ r = -3$

$T_8 = 2(-3)^7$

$T_8 = 2\ (-2187)$

$\underline{\underline{T_8 = 4374}}$

b) Evaluate 12th term of geometric progression 256, 218, 64…….

Solution

$\underline{T_{12} = ?}$

$T_{12} = ar^{11}$

$* \ a = 256 \quad * \ r = \frac{1}{2}$

$T_{12} = 256\left(\frac{1}{2}\right)^{11}$

$T_{12} = 256\frac{(1)^{11}}{(2)^{11}}$

$T_{12} = \frac{256 \times 1}{2048}$

$\underline{\underline{T_{12} = \frac{1}{8}}}$

Exercise 01

a) Evaluate (i) 5th term (ii) 10th term of a geometric progression 3, 6, 12……

b) Evaluate 7th term of a geometric progression 729, -243, 81, -27……

Example ②

Second and fifth terms of a geometric progression is 4 and 256 respectively.

(i) Evaluate first term and common ratio

(ii) Write down first four terms of a series

(iii) Calculate 7th term

(iv) Find the sum of first 6 terms of a series

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Solution

(i) Evaluate 'a' and 'r'

Given that,

$T_2 = 4$ and $T_5 = 256$

But $T_2 = ar$ $T_5 = ar^4$

$ar = 4$ ①

$ar^4 = 256$ ②

$\frac{②}{①}$ $\frac{ar^4}{ar} = \frac{256}{4}$

$r^3 = 64$

$r^3 = (4)^3$

$r = 4$

Sub r = 4, to ①

$ar = 4$

$4a = 4$

$a = 1$

(ii) First four terms

1, 4, 16, 64……

(iii) $T_7 = ?$

$T_7 = ar^6$

$* a = 1$ $* r = 4$

$T_7 = 1(4)^6$

$T_7 = 1 \times 4096$

$T_7 = 4096$

(iv) $S_6 = ?$

$S_n = \frac{a\{r^n - 1\}}{r - 1}$

$* a = 1$ $* r = 4$ $* n = 6$

$S_6 = \frac{1\{(4)^6 - 1\}}{4 - 1}$

$S_6 = \frac{1\{4096 - 1\}}{3}$

$S_6 = \frac{4095}{3}$

$S_6 = 1365$

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Exercise 02

Third and seventh terms of a geometric progression is 12 and 192 respectively.

(i) Evaluate first term and common difference
(ii) Write down first four terms of a series
(iii) Calculate 10^{th} term
(iv) Find the sum of first 12 terms of a series

Exercise 03

The second term of a geometric progression is greater than first term by 8. The sum of second and third terms is 48. Show that there are two progressions satisfying the above requirement and write down first four terms of each series.

Example ③

Income from a mine decreases every year by 12% of the income it generated in the previous year. The income in first year was 720 million rupees. Calculate,

(i) The income in 8^{th} year
(ii) The total income during first 8 tears.
(iii) The total income that would be expected in an infinitely longer period of time.

Solution

Series is as follows.

Year I $= 720 \qquad = a$
Year II $= 720\ (0.88) \qquad = a$
Year III $= 720\ (0.88)^2 \qquad = ar^2$

The above one is a GP with $a = 720$ and $r = 0.88$

(i) Income in 8^{th} year $= T_8$

$T_8 \quad = ar^7$
$a = 720 \quad r = 0.88$
$T_8 \quad = 720\ (0.88)^7$
$T_8 \quad = 294.25$ Million Rupees

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(ii) Total Income during first 8 years = S_8

$$S_n = \frac{a\{r^n - 1\}}{r - 1}$$

$* a = 720 \quad * r = 0.88 \quad * n = 8$

$$S_8 = \frac{720\{(0.88)^8 - 1\}}{0.88 - 1}$$

$$S_8 = \frac{720\{0.36 - 1\}}{-0.12}$$

$$S_8 = \frac{720(-0.64)}{-0.12}$$

S_8 = 3840 Million Rupees

(iii) Total Income expected infinite longer period of time = S_{00}

r = 0.88 ($-1<r<+1$) S_{00} valid

$$S_{00} = \frac{a}{1 - r}$$

$$S_{00} = \frac{720}{1 - 0.88}$$

$$S_{00} = \frac{720}{0.12}$$

S_{00} = 6000 Million Rupees

Exercise 03

A person saves every month 10% more than the amount se saved in previous month. His saving in first month was Rs. 800.

(i) Find the amount he saved in 7^{th} month.
(ii) Find the amount he saves during first 12 months.