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Basic Concepts of Probability

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Introduction

Mathematical value that can be given for an uncertainty can be described as probability. As an example suppose that we toss a fair coin at once, there is only two possible outcomes as head and tail. We can say that there is a 50% chance of getting head and another 50% chance of getting tail.

Basic Definitions Involving in Probability

Random Experiment

Any process that generates a set of observation or out comes can be described as a random experiment.

Examples

- Tossing a fair coin at once.
- Arrival of customers at a service station.
- No of defectives items produced by a machine.

Sample Space

Set of all possible outcomes from a random experiment is a sample space.

Examples

- Consider the experiment of tossing a six-sided die at once. Sample space E is given as follows.

  $E\text{-}\{1,2,3,4,5,6\}$

- Consider the experiment of tossing two fair coins at once.

  $E=\{(HH),(HT),(TH),(TT)\}$

Event

Event is a subset of a sample space.

Example

Consider the experiment of tossing a six sided die at once. We can define following from the above experiment.

E={1,2,3,4,5,6}

A={Getting 1 on the upper face}={1}

B={Getting an even number on the upper face}={2,4,6}

C={Getting a number less than 5}={1,2,3,4}

D={Getting 7 on the upper face} = { }

Q={Getting a number less than or equal 6}={1 2 3 4 5 6}

Events are of two types

- Simple event
- Compound event

Simple Event

If an event consists only one sample point, then it is a simple event or elementary event. The above example A is a simple event.

Compound Event

If an event having only two of more sample points, then it is compound event. The above example B and C are compound events.

Mathematical Definition of A Probability

If E is a sample space in an experiment and S is a subset of E, then the probability of occurring event S can be described as follows.

$$\boldsymbol{P(s) = \frac{n(s)}{s(E)}}$$

P(s) = Probability of occurring Event S

n(s) = No. of elements in set 'S'

n(E) = No. of elements in a sample space

Note

- Probability is a value in between 0 and 1 both inclusive

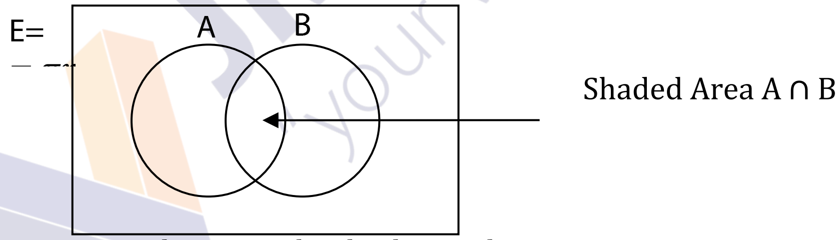
$(0 \leq P \leq 1)$

- Sum of all probabilities of simple events is equal to 1

$\sum P = 1$

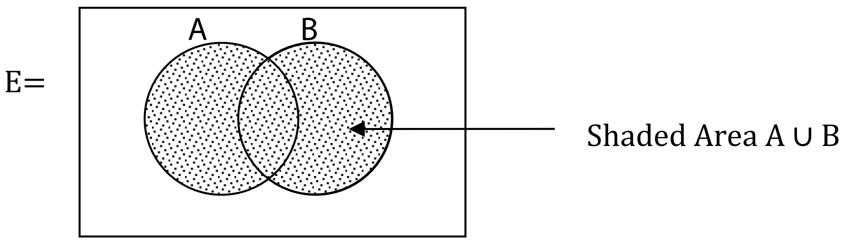
Event Relations

The Event A∩B



A∩B represent the event that both A and B occurs at once.

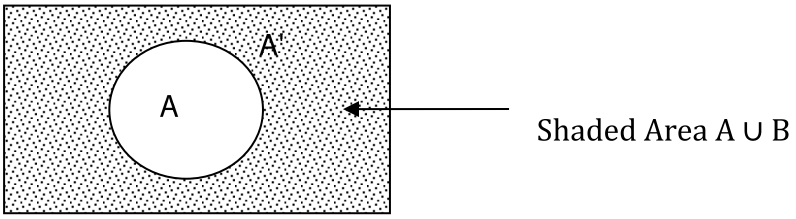
The Event A ∪ B



A∪B represents the event that either A or B occurs (At least one event)

The Event A'

E=



The event A' represents the event that close not occur an event A

Example 1

Two six sided dice thrown together.

(i) Write down the sample space

(ii) Evaluate probability that

a) Equal scores on both upper faces

b) Sum of the scores on upper faces is > 9

Example 2

A card is selected at random from a pack of 52 playing cards. C is the event the card selected is a club and K is the event that card selected is a king. Calculate following probabilities.

(i) $P(K)$

(ii) $P(C)$

(iii) $P(K \cap C)$

(iv) $P(K' \cap C)$

(v) $P(K \cap C')$

(vi) $P(C')$

Example 3

There are 25 students in a particular class. There are 16 students Studying German and 14 studying French and 6 students studying both languages. Find the probability that randomly selected student from the above class studying.

i. French
ii. French and German
iii. French but not German
iv. Does not study both languages

ADDITIONAL RULE OF PROBABILITY

Let A and B are any two events defined an sample space E

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 1

The probability that a student get price for mathematics is 0.44 and a probability that student will get price for statistics is 0.58. It is also found that the probability of receiving both prices is 0.16. Find the probability that student will receive at least one of these prices.

Example 2

A and B are two events that, and. Find

(i) $P(A \cap B)$

(ii) $P(A')$

(iii) $P(A' \cap B)$

(iv) $P(A' \cup B)$

Example 3

In a particular company there are 100 employees under following categorization.

	Male	Female	Total
Executive	20	12	32
Non-executive	50	18	68
Total	70	30	Grand Total = 100

If one employee selected random from the above company, calculate the probability that the employee is

(i) A male

(ii) An executive

(iii) A female executive

(iv) A male non-executive

(v) Either male or executive

(vi) Either female or non-executive

Multiplication Rule of Probability

Let A and B are any two events defined on sample space E

$$P(A \cap B) = P(A)P\left(\frac{B}{A}\right)$$

$P\left(\frac{B}{A}\right)$ = Probability of B, given A

(Conditional Probability)

Conditional Probability

From a multiplication rule

$$P(A \cap B) \quad = \quad P(A)P\left(\frac{B}{A}\right)$$

$$P\frac{(A \cap B)}{P(A)} \quad = \quad P\left(\frac{B}{A}\right)$$

Therefore probability of B, given A can be defined as:

$$P\left(\frac{B}{A}\right) = \frac{(A \cap B)}{P(A)} \qquad P(A) \neq 0$$

In the same manner probability of A given B can be defined as

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \qquad P(B) \neq 0$$

Tree Diagram

Consider the following example. Suppose that there are 5 Res and 3 Blue marbles in a box. A marble is drawn and do not replace it. Then second marble is drawn.

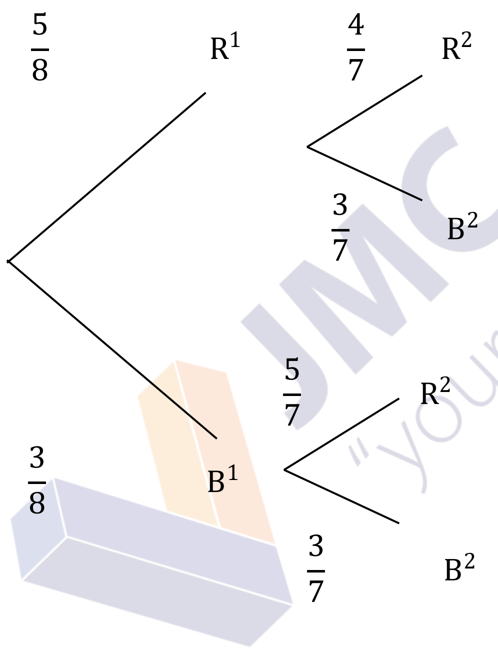
Let us define following events.

R1 - First marble is Red

B1 - First marble is Blue

R2 - Second marble is Red

B2 - Second marble is Blue



Following probabilities available

$P(R_1) = \frac{5}{8}$

$P(B_1) = \frac{3}{8}$

Conditional Probabilities

$P\left(\frac{R_2}{R_1}\right)=\frac{4}{7}$ {Second marble is Red given that First marble is Red}

$P\left(\frac{B_2}{R_1}\right)=\frac{3}{7}$ {Second marble is Blue given that First marble is Red}

$P\left(\frac{R_2}{B_2}\right)=\frac{5}{7}$ {Second marble is Red given that First marble is Blue}

$P\left(\frac{B_2}{B_1}\right)=\frac{2}{7}$ {Second marble is Blue given that First marble is Blue}

Evaluate following probabilities by using above tree diagram.

(i) Both marbles are Red

(ii) Marbles are in same colour

(iii) Second marble is Red

(iv) First marble is Blue given that second marble is Red.

Answer

(i) Both marbles are Red

$P(R_1 \cap R_2)=P(R_1)P\left(\frac{R_2}{R_1}\right)$

$=\frac{5}{8}\times\frac{4}{7}$

$P(R_1 \cap R_2)=\left(\frac{20}{56}\right)$

(ii) Marbles are in Same Colour

$P(R_1 \cap R_2)+P(B_1 \cap B_2)$

$P(R_1)P\left(\frac{R_2}{R_1}\right)+P(B_1)P\left(\frac{B_2}{B_1}\right)$

$\left(\frac{5}{8}\times\frac{4}{7}\right)+\left(\frac{3}{8}\times\frac{2}{7}\right)$

$\frac{20}{56}+\frac{6}{56}$

$P(R_1 \cap R_2)+P(B_1 \cap B_2)=\frac{26}{56}$

(iii) Second marble is Red

$$P(R_2)=P(R_1 \cap R_2)+P(B_1 \cap R_2)$$

$$=P(R_1)P(R_2/R_1)+P(B_1)P(R_2/B_1)$$

$$P(R_2)=\left(\frac{5}{8}\times\frac{4}{7}\right)+\left(\frac{3}{8}\times\frac{5}{7}\right)$$

$$P(R_2)\frac{20}{56}+\frac{15}{56}$$

$$P(R_2)=\frac{35}{56}$$

(iv) First marble is Blue given that second marble is Red

$$P=\left(\frac{B_1}{R_2}\right)=P\left(\frac{B_1 \cap R_2}{P(R_2)}\right)$$

$$=\left(\frac{3}{8}\times\frac{5}{7}\right)\div\frac{35}{56}$$

$$=\frac{15}{\cancel{56}}\times\frac{\cancel{56}}{35}$$

$$P\ =\left(\frac{B_1}{R_2}\right)=\frac{15}{35}$$

Example 1

There are 6 White and 4 Black marbles in a box. A marble is drawn and do not replace it. Then second marble is drawn

a) Draw a tree diagram for the above experiment and describe all probabilities

b) Evaluate following probabilities

(i) Both marbles are White

(ii) Marbles are in same colour

(iii) Second marble is Black

(v) First marble is Black, given that second marble is also Black

Example 2

A and B are bottle sealing machines which are used to sealed soft drink bottles in a production line. Machine a sealed 70% and machine B sealed 30% from the total production. It is found that 5% of the bottles sealed by machine A and 3% of the bottles sealed by machine B were defective.

(i) If we select a sealed bottle randomly, what is the probability that it is defective?

(ii) If we select a defective bottle randomly, what is the probability that it was sealed by

a) Machine A

b) Machine B

Example 3

Ananda, Ravi and Malik are engaged in washing dishes in restaurant. Since Ananda is oldest he washes 40% of the time, Ravi and Malik each washes 30% of the time.

When Ananda washes the probability of at least one dish being broken is 0.01, when Ravi Washes the probability is 0.02 and when Malik washes the probability is 0.03.

It is not known who is washing the dishes on.

Example 4

The staff in an organization is divided into 3 categories as follows.

	Male (M)	Female (F)
Administrative Staff (A)	20	30
Executive Staff (E)	60	140
Sales Staff (S)	100	50

If staff member selected randomly, Find the probability that staff member is

(i) A female

(ii) A male in an executive staff

(iii) Either female or member in an administrative staff

(iv) An administrative staff member given that he is a male.

(v) A female given that she is a sales staff member.

Independent Events

Two or more events are said to be independent if the probability of occurrence of anyone of these events not change by the occurrence of any other remaining events.

Let A and B are independent events defined on sample space E.

Multiplication Rule

$$P(A \cap B) = P(A)P(B)$$

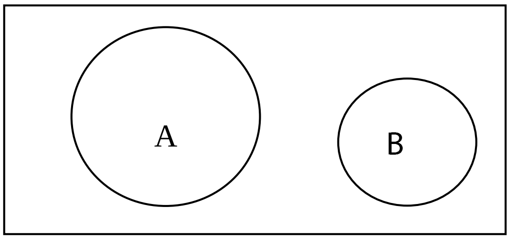
Additional Rule

$$P(A \cup B) = P(A) + P(B) - P(B)$$

Mutually Exclusive Events

Two or more events does not occur at once, we say that events are mutually exclusive. It can be represented by using following Venn-Diagram.

E=



Let A and B are mutually exclusive events defined on sample space E.

Multiplication Rule

$$P(A \cap B) = 0$$

Additional Rule

$$P(A \cup B) = P(A) + P(B)$$

Example 1

Suppose that there are 5 Red and 3 Blue marbles in a box. A marble is drawn and replaced it. Then second marble is drawn.

a) Draw a tree diagram for the above experiment
b) Evaluate probability that
   (i) Both marbles are Red
   (ii) Marble are in same colour
   (iii) At least one Red marble

Probability Distributions

Random Variable

A random variable is a function that assigns a numeric value to each simple event on a sample space.

Example 1

Consider the experiment of tossing a six sided die at once.

$E = \{1,2,3,4,5,6\}$

Let the random variable X as:

X = Value on the upper face

$E = \{X/X = 1,2,3,4,5,6\}$

Example 2

Consider the experiment of tossing a fair coin in three times.

$E = \{(HHH),(HHT),(HTH),(THH),(TTH),(THT),(HTT),(TTT)\}$

Let random variable X as

X = No. of heads

$E = \{X/X = 0,1,2,3\}$

Expected Value

Mean of a probability distribution with decrees random variable is defined as expected value and it is denoted by . Expected value can be evaluate by using following formula.

$$E(X)=\sum x\,P(x)$$

Variance

Variance of probability distribution with a discrete random variable can be evaluate by using following formula.

$$\mathrm{Var}(x)=\sum x^2P(x)\ -\{E(x)\}^2$$

$$\text{Standard Deviation}=\sqrt{\mathrm{Var}(x)}$$

Example 1

Grade points and the probabilities in on accountancy course is given below. Calculate expected value and variance for the data.

Grade	Grade Point	Probability
A	04	0.12
B	03	0.32
C	02	0.38
D	01	0.15
E	00	0.03

Example 2

The discrete random variable X has a probability function

$P(x)=K(1-x)$ $X=0,1$

$=K(x-1)$ $X=2,3$

$=0$ Otherwise

(i) Show that

$$K=\frac{1}{4}$$

(ii) Evaluate expected value and variance

Example 3

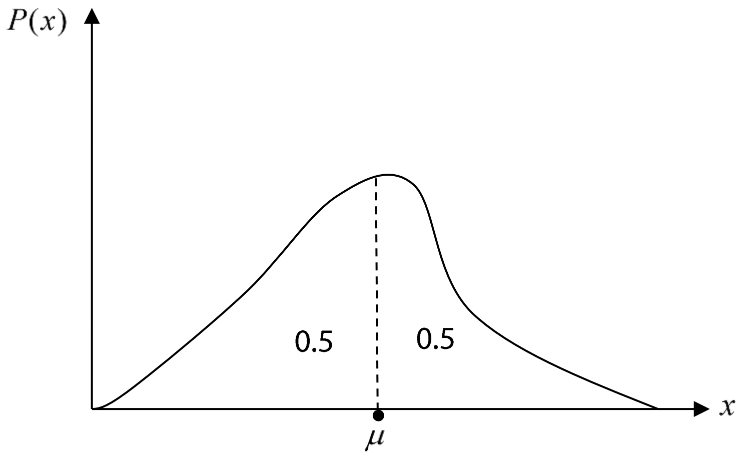
The random variable X has following probability distribution.

X	1	2	3	4	5
P(x)	0.1	P	0.3	q	0.2

Given that expected value is 3 determine constants p and q.

Probability Distribution Function of a Normal Distribution

Graph of a Normal Distribution

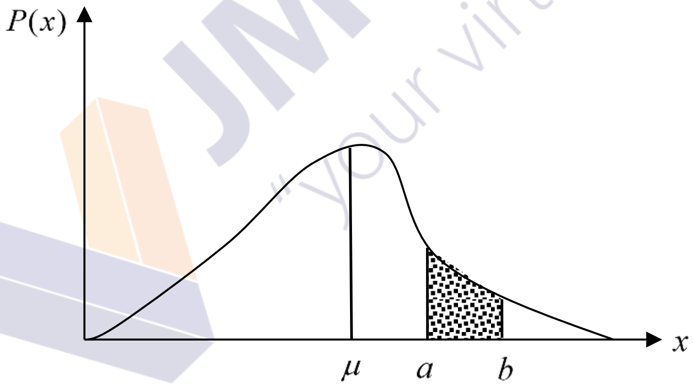


Properties of Normal Distribution Curve

- The curve is symmetrical
- Mean, median, mode has same value
- Total area under normal distribution curve is equal to 1
- Two ends of normal curve does not touch the X-axis.

Probability of a Normally Distribution Random Variable

Let X ~ N (μ, ϱ)



$$P(a<x<b)=\int_{c}^{b} P(x)\,dx$$

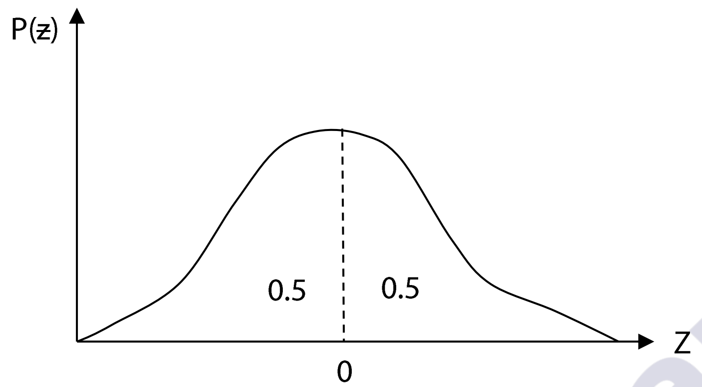
P=(a<x<b)=area of normal curve in between X=a and X=b

Standard Normal Distribution

Random variable with any mean and any standard deviation can be fitted into standard curve what is known as standard normal distribution. Random variable of a standard normal distribution is denoted by Z standard normal curve has following properties.

- It's mean is equal 0
- It's standard deviation is 1
- Total area under standard normal curve is equal to 1

$Z \approx N\ (0,1)$



A random variable X is normally distributed with mean and standard deviation can be transformed into standard normal mode by using following formula.

$$Z = \left(\frac{x-\mu}{\sigma}\right)$$

Example 1

Given that Z ~ N (0, 1) Evaluate following probabilities.

a) $P(Z > 1.0)$

b) $P(Z < -128)$

c) $P((-1.96 < Z < 2.33)$

d) $P(Z > -2.58)$

e) $P(Z < 1.65)$

Example 2

Given that X ~ N (50, 10) Evaluate following probabilities.

a) $P(x > 6)$

b) $P(x < 38)$

c) $P(25 < x < 58)$

d) $P(x > 23)$

e) $P(x < 76)$

Example 3

The measure of intelligence (IQ) of a group of students is assumed to be normally distributed with mean 100 and standard deviation 15. Find the probability that a student selected at random has an IQ

a) Less than 91

b) More than 80

c) In between 85 and 125

Example 4

Marks obtained by 800 candidates of an examination are approximately normally distributed with mean 56 and standard deviation 8.

a) If 2.5% of students obtained distinction by scoring more than X_0 marks, Find the value of X_0.

b) If 1% of candidates obtained weak passes by scoring less than W_0 marks find the value of W_0

Example 5

Marks obtained in an English test were normally distributed with mean μ and standard deviation σ 10% students scored more than 70 marks. 20% of students scored less than 40 marks. Find the values of μ and σ.